Physics of Fluids

UNIVERSITY OF TWENTE.

European Research Council
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NWO

J.M.Burgerscentrum
Research School for Fluid Mechanics

Melting of Cylindrical Laboratory Icebergs

Morphology, capsizing and plumes

Edoardo Bellincioni, WHOI - GFD Program, 1st August 2024

Supervised by

D. Lohse

S.G. Huisman

https://www.expeditiongreenland.com

www.CHASINGICE.com

Melting

- How does freely floating ice melt?

Rotational stability

- Why and how does it rotate?

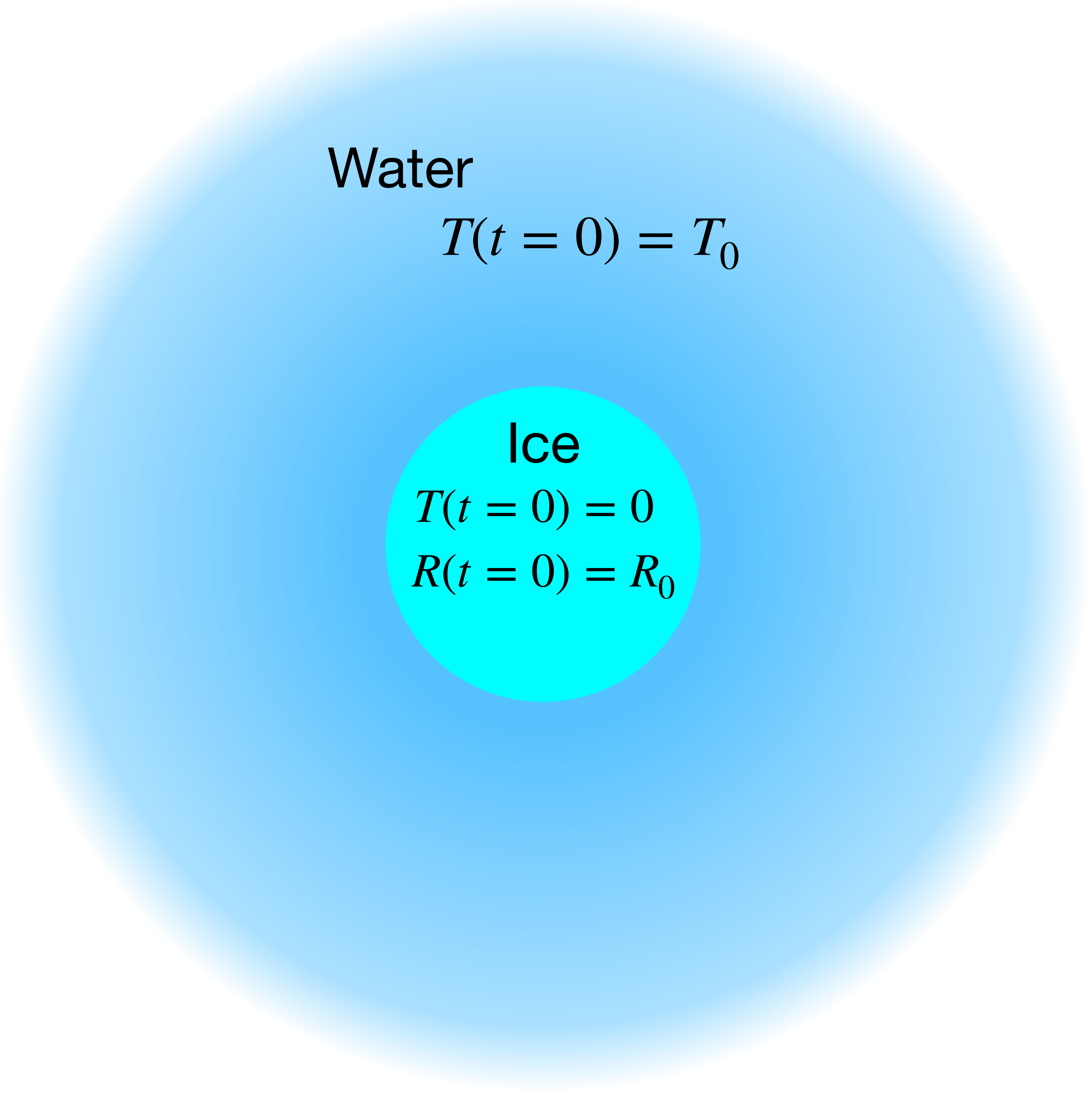
Interaction with surroundings

- In what fluid is the ice immersed? How is the fluid affected?

What to expect

A known solution

3D (sphere), no gravity



Temperature in the liquid

$$\partial_t T = \alpha \Delta T$$

With BC

$$T(r,0) = T_0\,, r > R$$

$$\lim_{r\to\infty} T(t,r) = T_0\,, t > 0$$

$$T(R(t),t) = 0\,, t > 0$$

Laplacian in 3D

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}$$

Scaled variable

$$\Theta = r\,T$$

New PDE

$$\frac{\partial \Theta}{\partial t} = \alpha \frac{\partial^2 \Theta}{\partial r^2}$$

With BC

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A known solution

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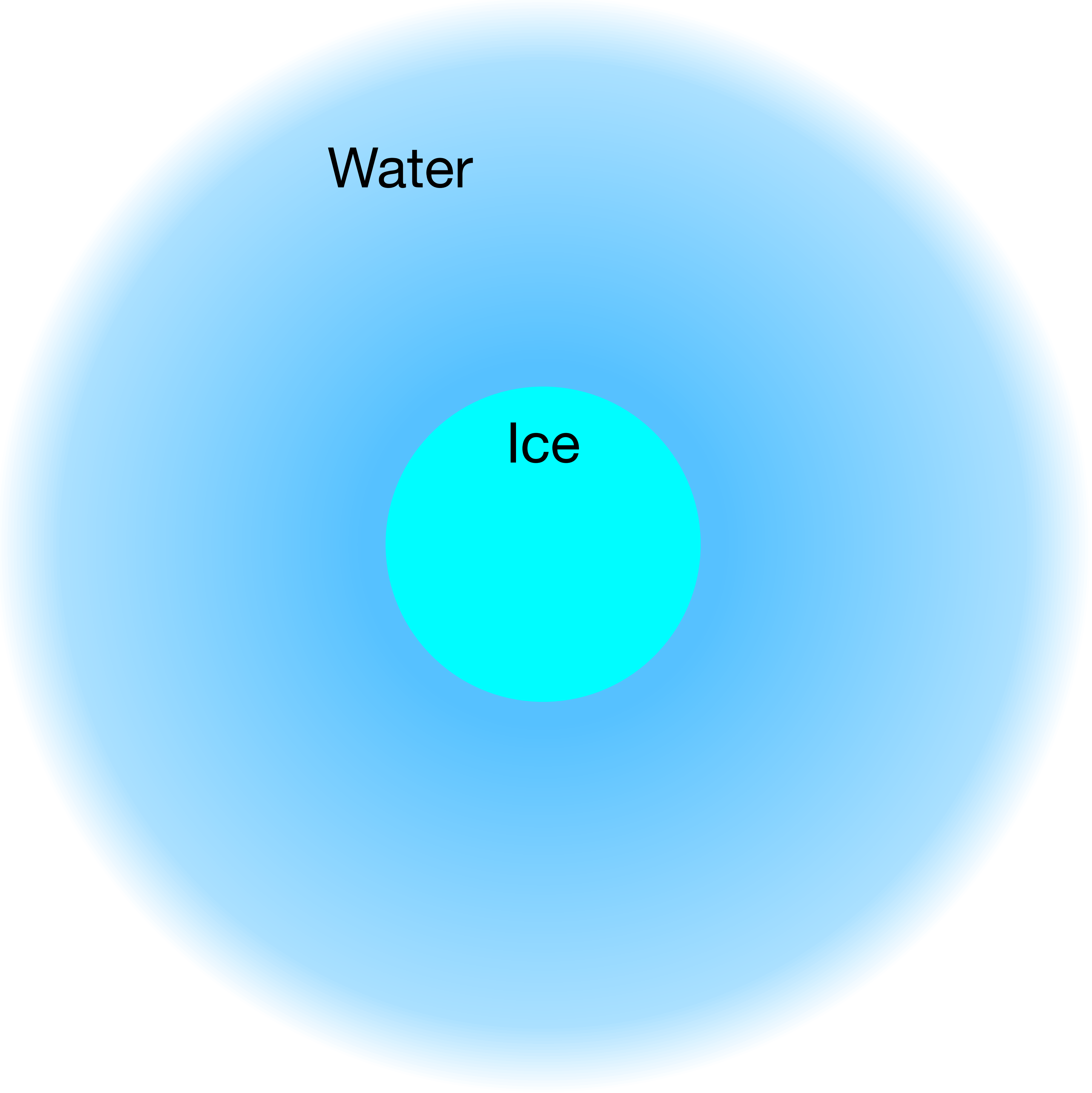


New PDE

$$\frac{\partial \Theta}{\partial t} = \alpha \frac{\partial^2 \Theta}{\partial r^2}$$

With BC

$$\Theta(r,0) = r\,T_0$$
$$\Theta(R(t), t) = 0$$

Solution (known)

$$\Theta(r,t) = \frac{T_0}{2\sqrt{\pi\alpha t}} \int_0^\infty (R+\xi')\{exp[-(r-R-\xi')^2/4\alpha t] - exp[-(r-R+\xi')^2/4\alpha t]\}d\xi'$$

Gradient at the boundary

$$\left.\frac{\partial \Theta}{\partial r}\right|_{r=R} = T_0\left\{1 + \frac{R}{\sqrt{\pi\alpha t}}\right\}$$

Thus

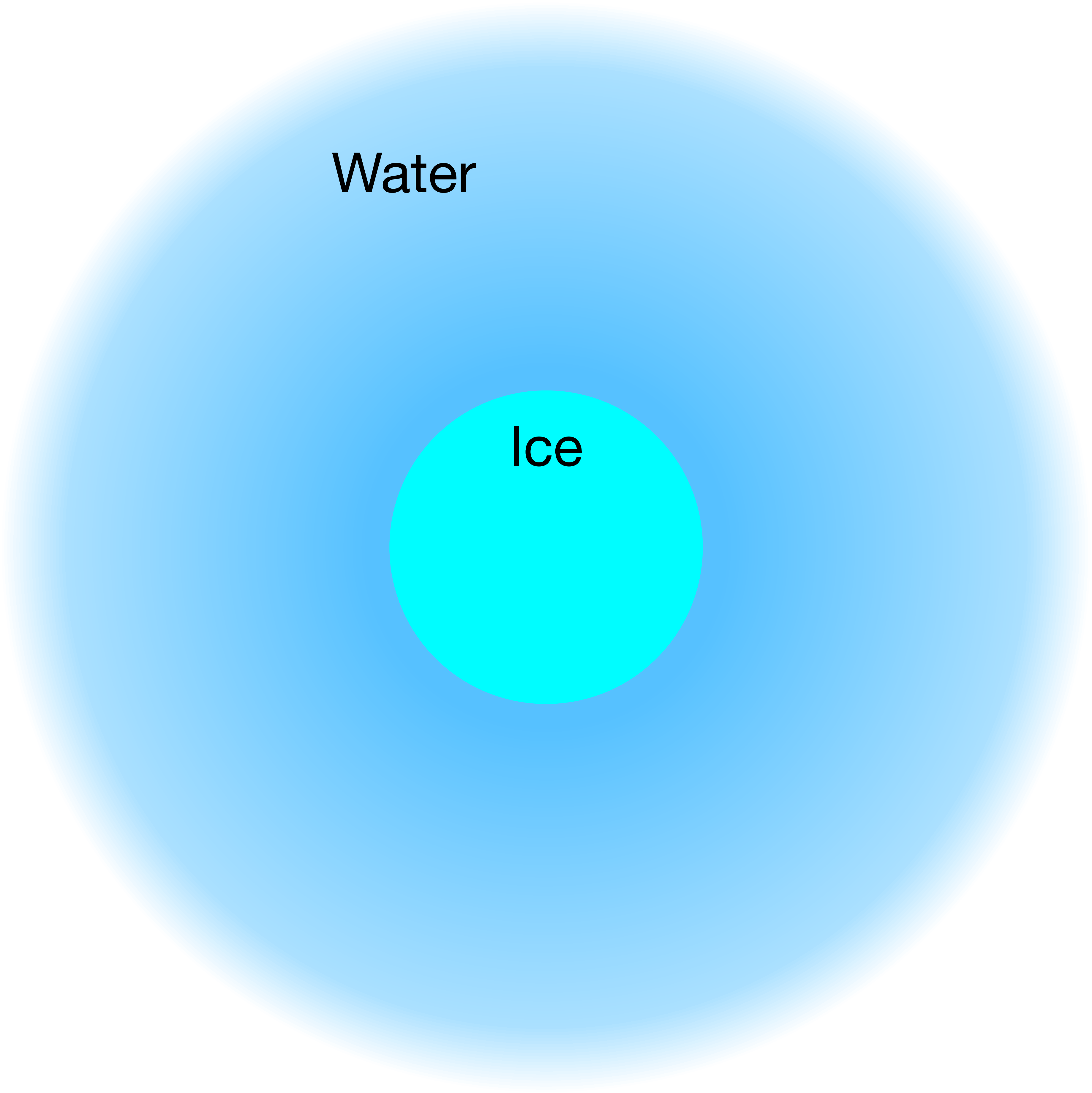
$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = T_0\left\{\frac{1}{R} + \frac{1}{\sqrt{\pi\alpha t}}\right\}$$

Applying heat balance at the boundary (Stefan condition)

$$\frac{dm}{dt}\mathscr{L} = 4\pi R^2 k \left.\frac{\partial T}{\partial r}\right|_{r=R}$$
$$= 4\pi R^2 k T_0\left\{\frac{1}{R} + \frac{1}{\sqrt{\pi\alpha t}}\right\}$$

A known solution

3D (sphere), no gravity



$$\frac{dm}{dt}\mathscr{L} = 4\pi R^2 k T_0 \left\{ \frac{1}{R} + \frac{1}{\sqrt{\pi \alpha t}} \right\}$$

Considering the mass loss at the surface of a sphere

$$\frac{dm}{dt} = 4\pi R^2 \rho_s \frac{dR}{dt}$$

Putting everything together

$$\frac{dR}{dt} = \frac{kT_0}{\rho_s \mathscr{L}} \left\{ \frac{1}{R} + \frac{1}{\sqrt{\pi \alpha t}} \right\}$$

THE JOURNAL OF CHEMICAL PHYSICS VOLUME 18, NUMBER 11 NOVEMBER, 1950

On the Stability of Gas Bubbles in Liquid-Gas Solutions

P. S. Epstein and M. S. Plesset
California Institute of Technology, Pasadena, California

(Received July 31, 1950)

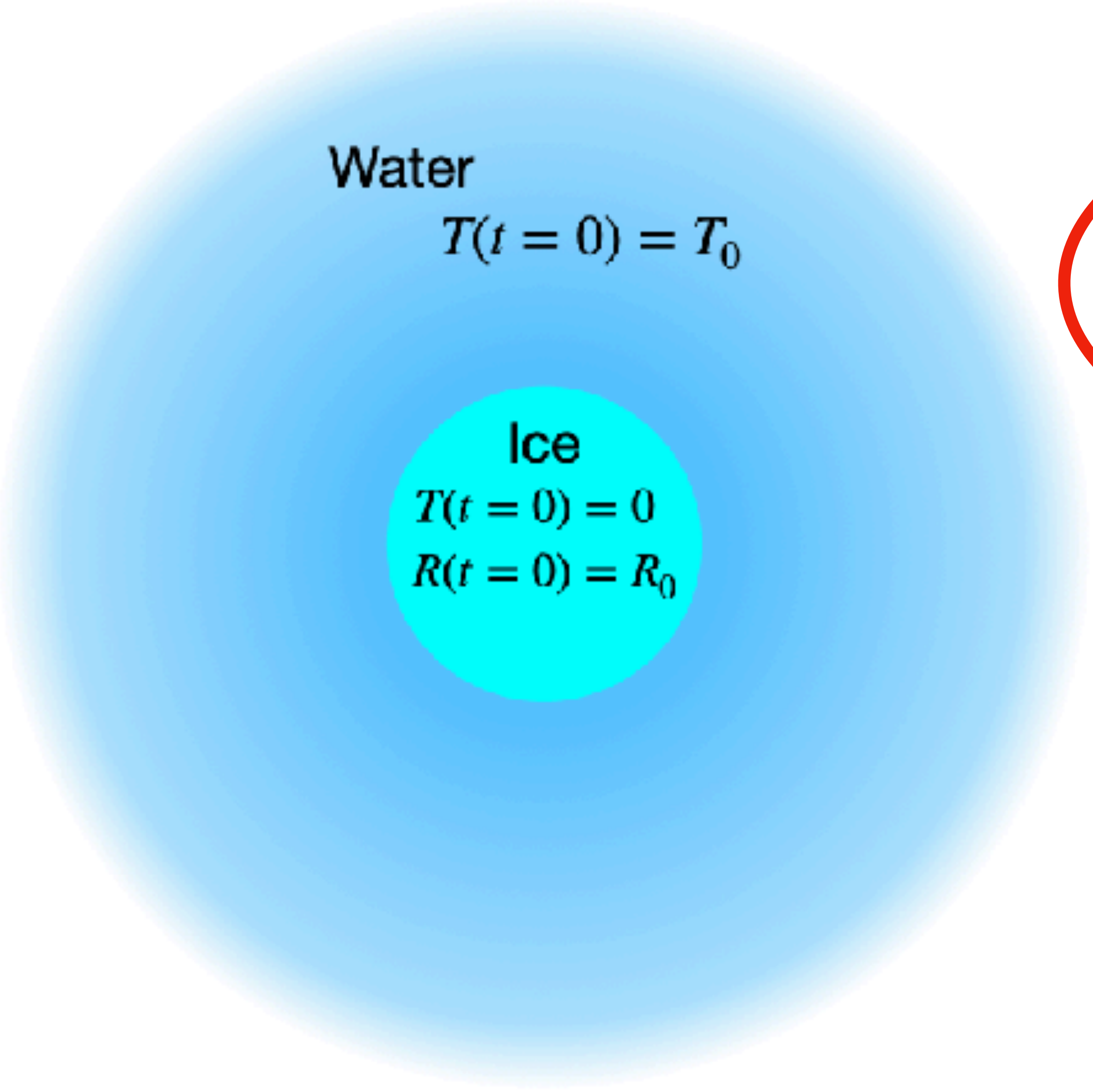
With the neglect of the translational motion of the bubble, approximate solutions may be found for the rate of solution by diffusion of a gas bubble in an undersaturated liquid-gas solution; approximate solutions are also presented for the rate of growth of a bubble in an oversaturated liquid-gas solution. The effect of surface tension on the diffusion process is also considered.

In 2D?

No luck whatsoever

A known solution

3D (sphere), no gravity



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With BC

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$$\Theta(R(t),t) = 0$$

Further (neglected) complications

Advection

Convection

Conduction in solid

Solutes

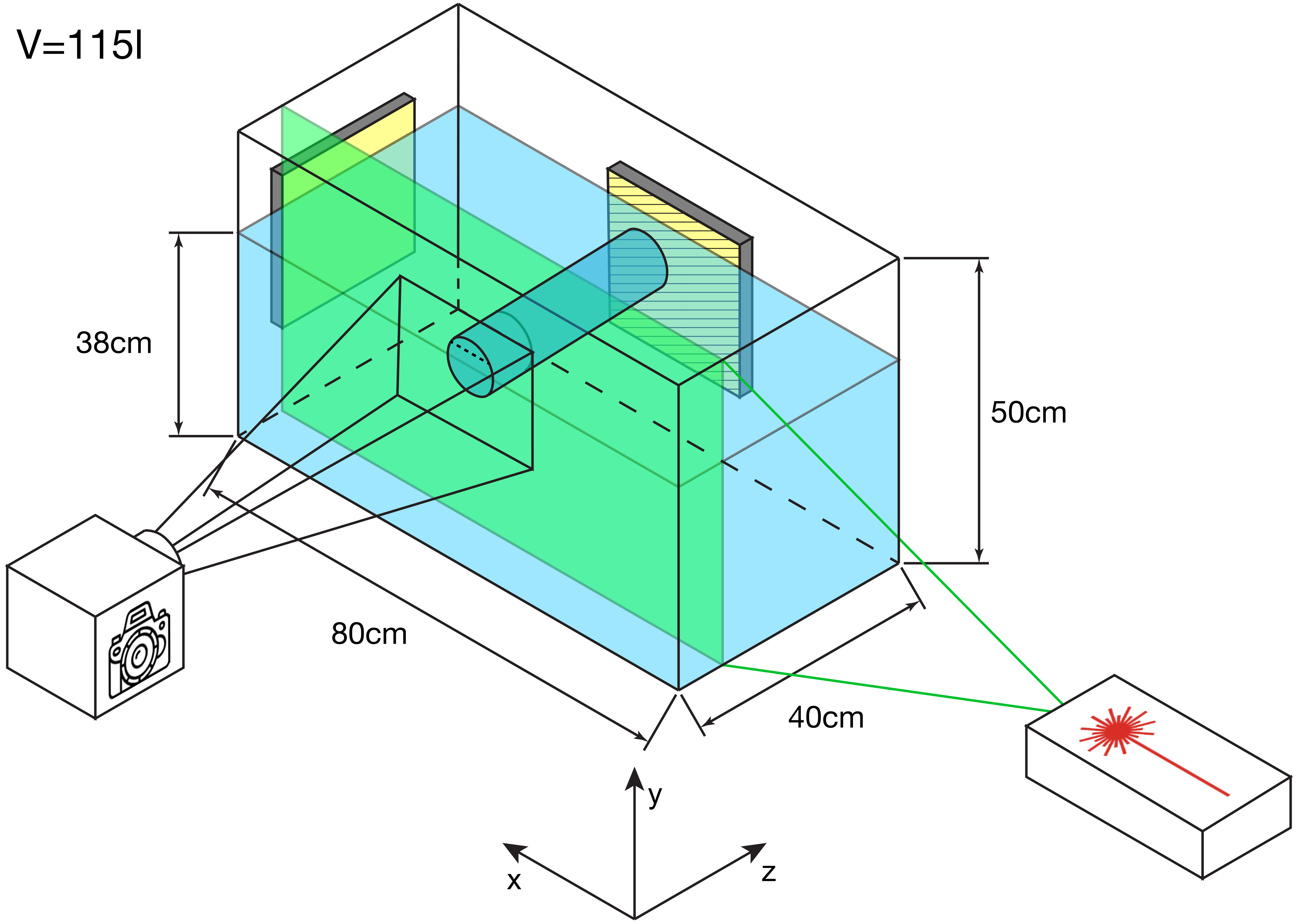
...

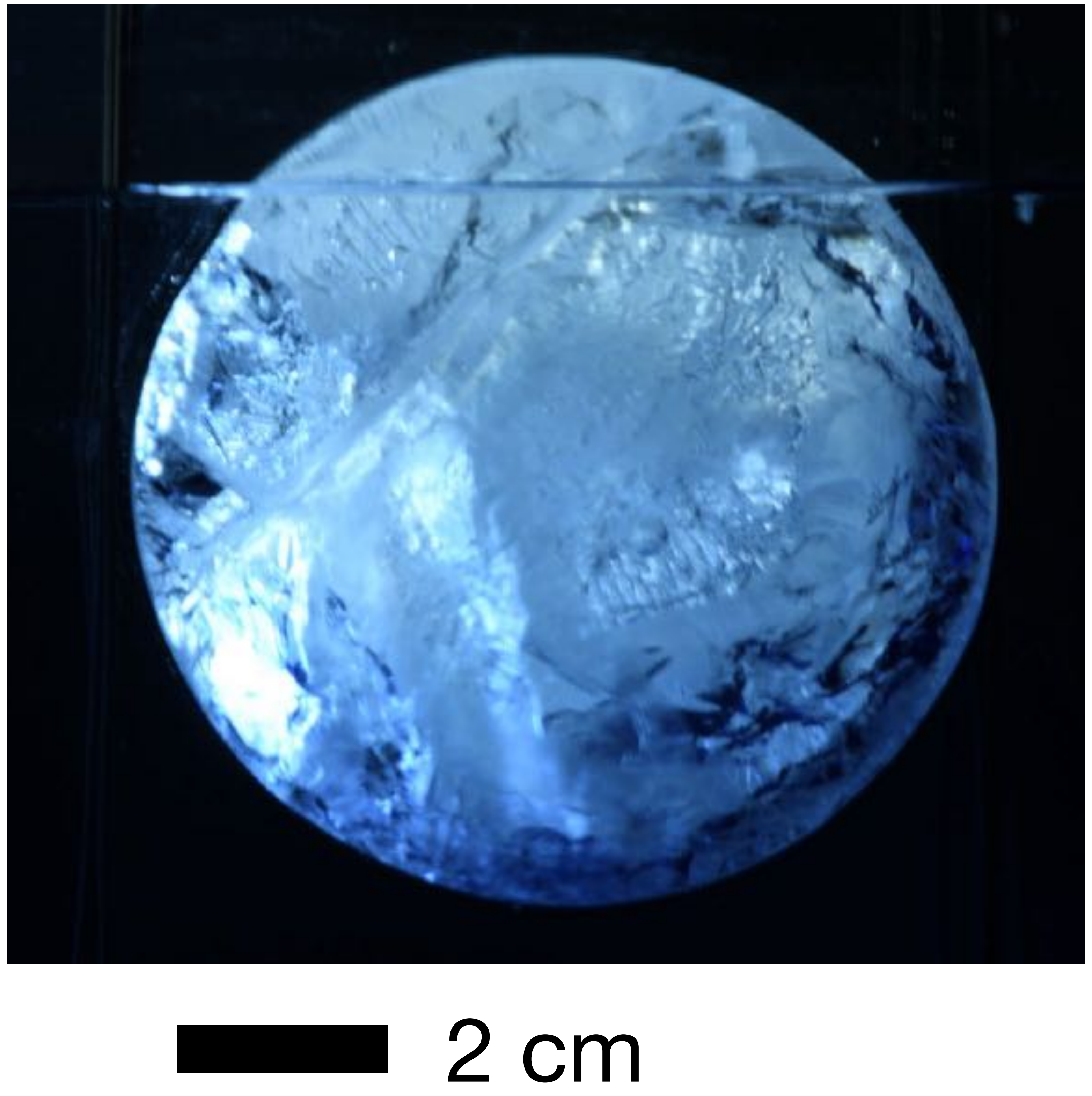
The Experiment

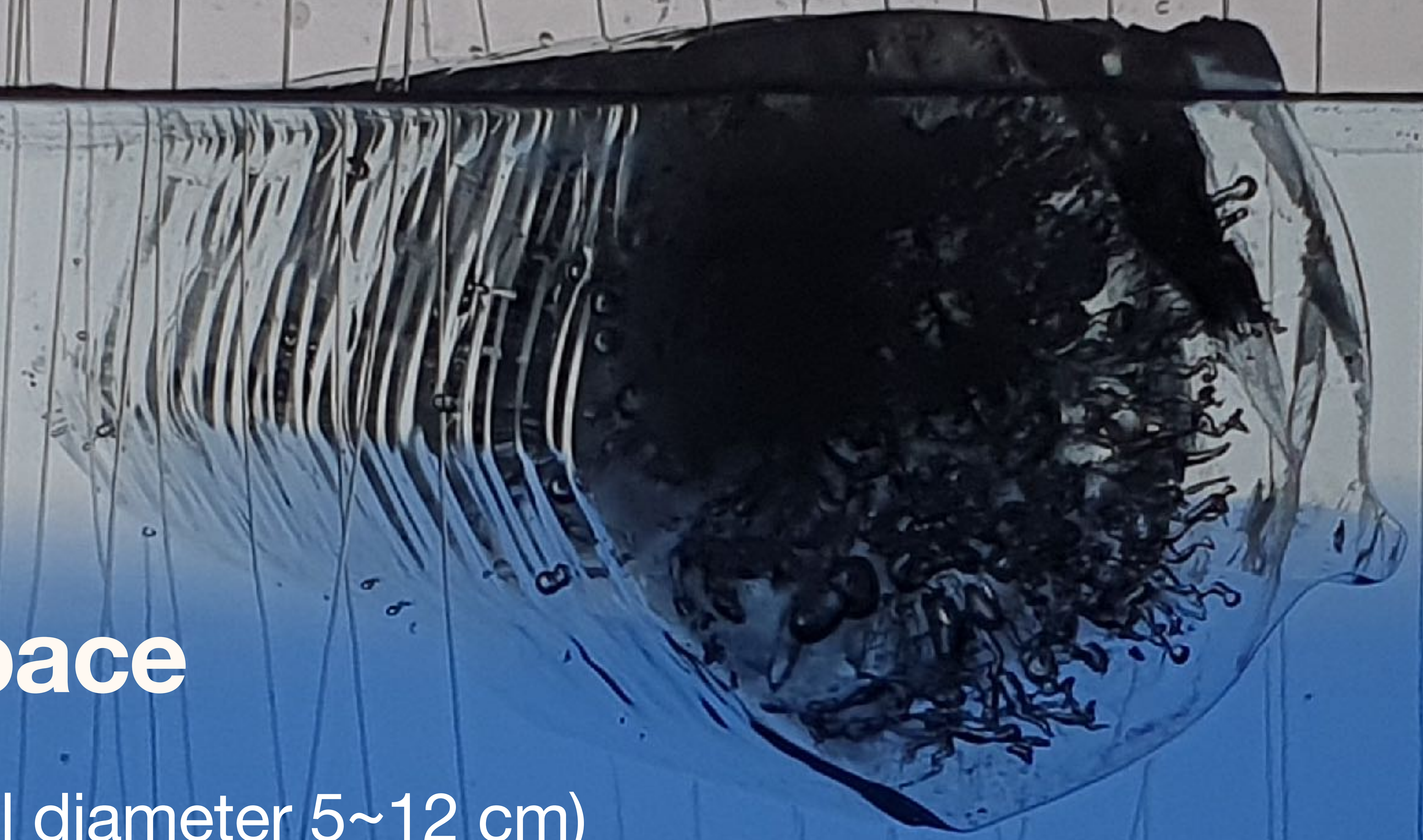
Why a cylinder?

Experimental setup

Tank, lighting and imaging



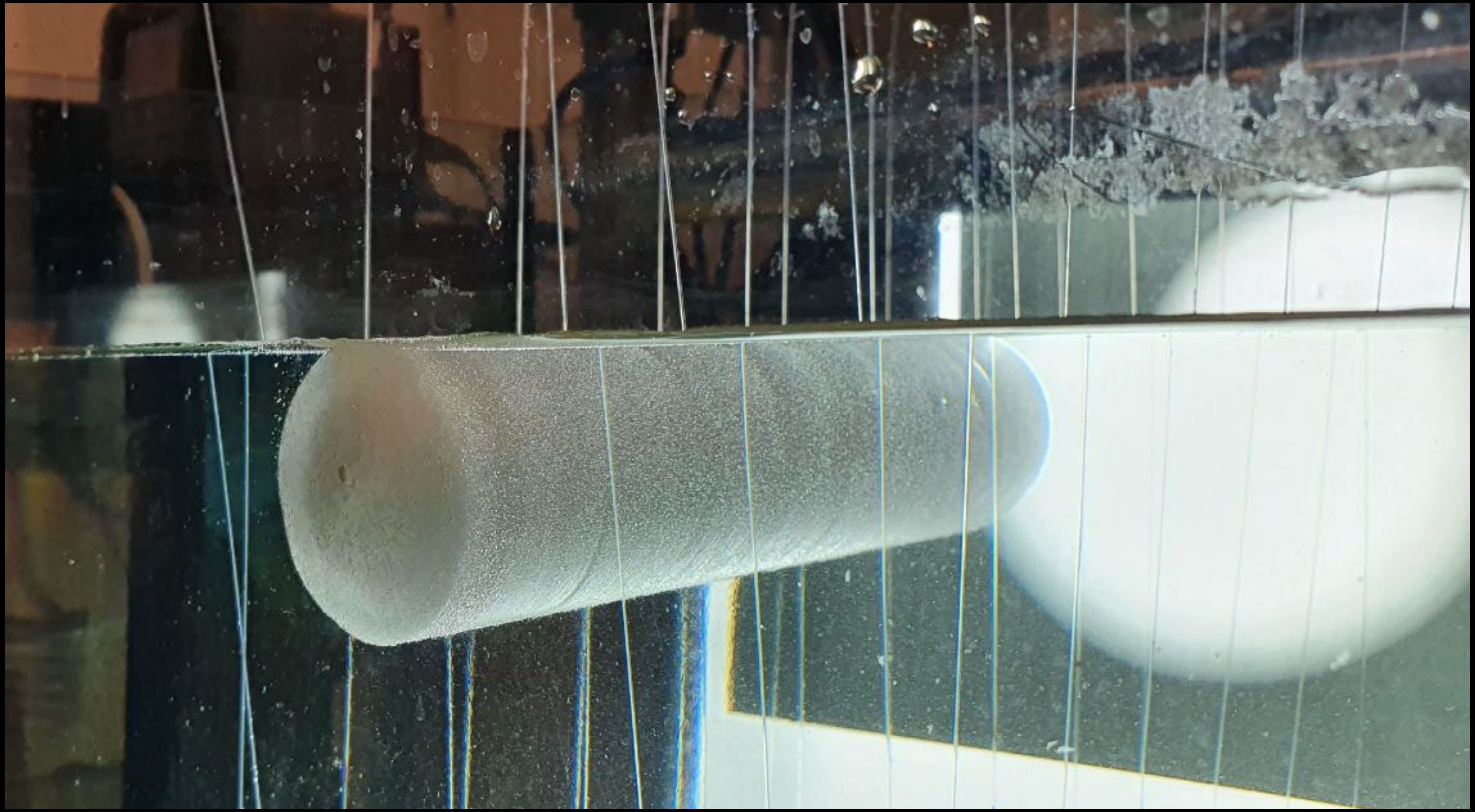




Parameter space

- Cylinder size (initial diameter 5~12 cm)
- Salinity of bulk water: 0, 10, 35 g/l + stratified
- Heavy water cylinder: density matched

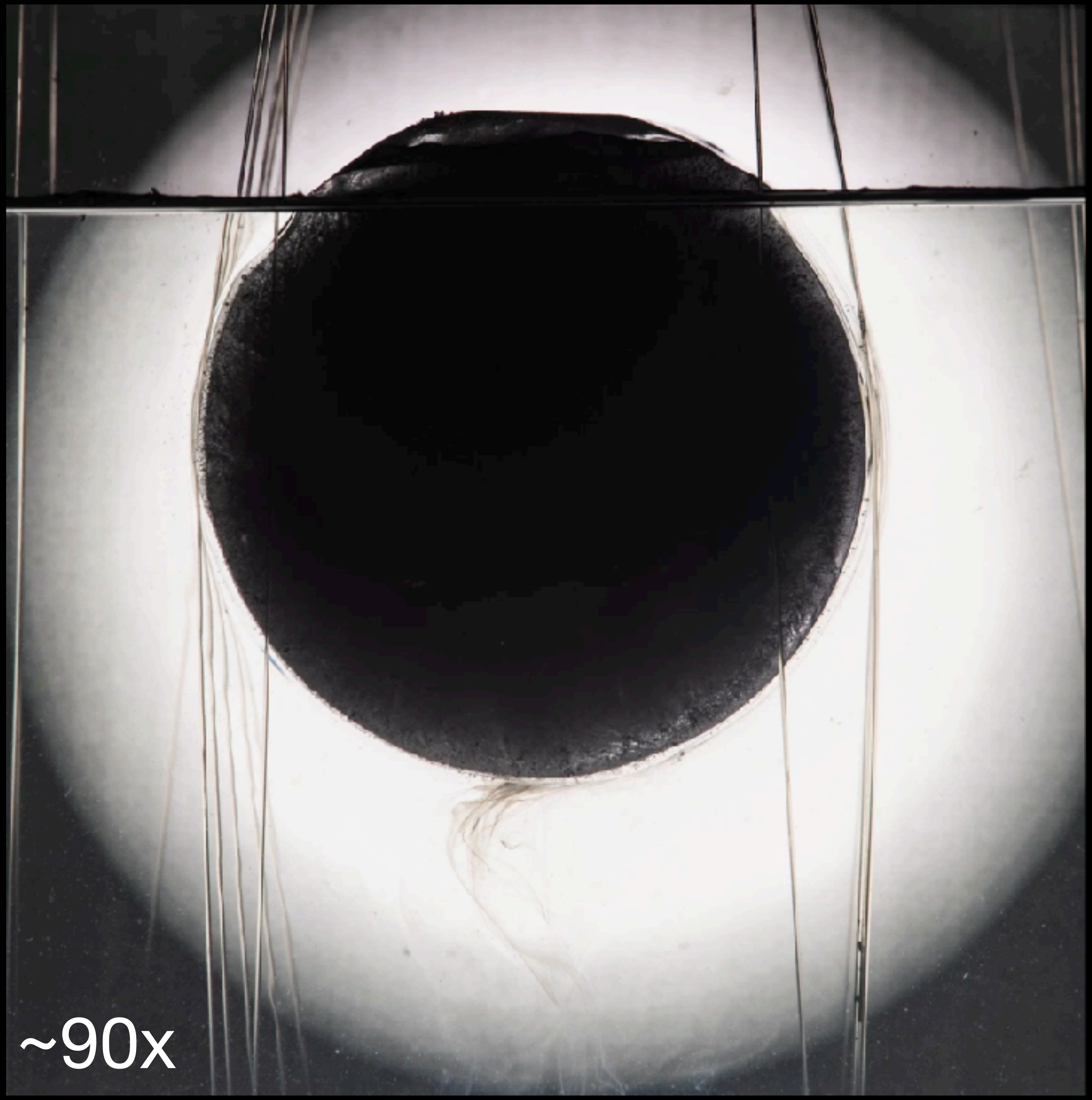
A non-floating ice

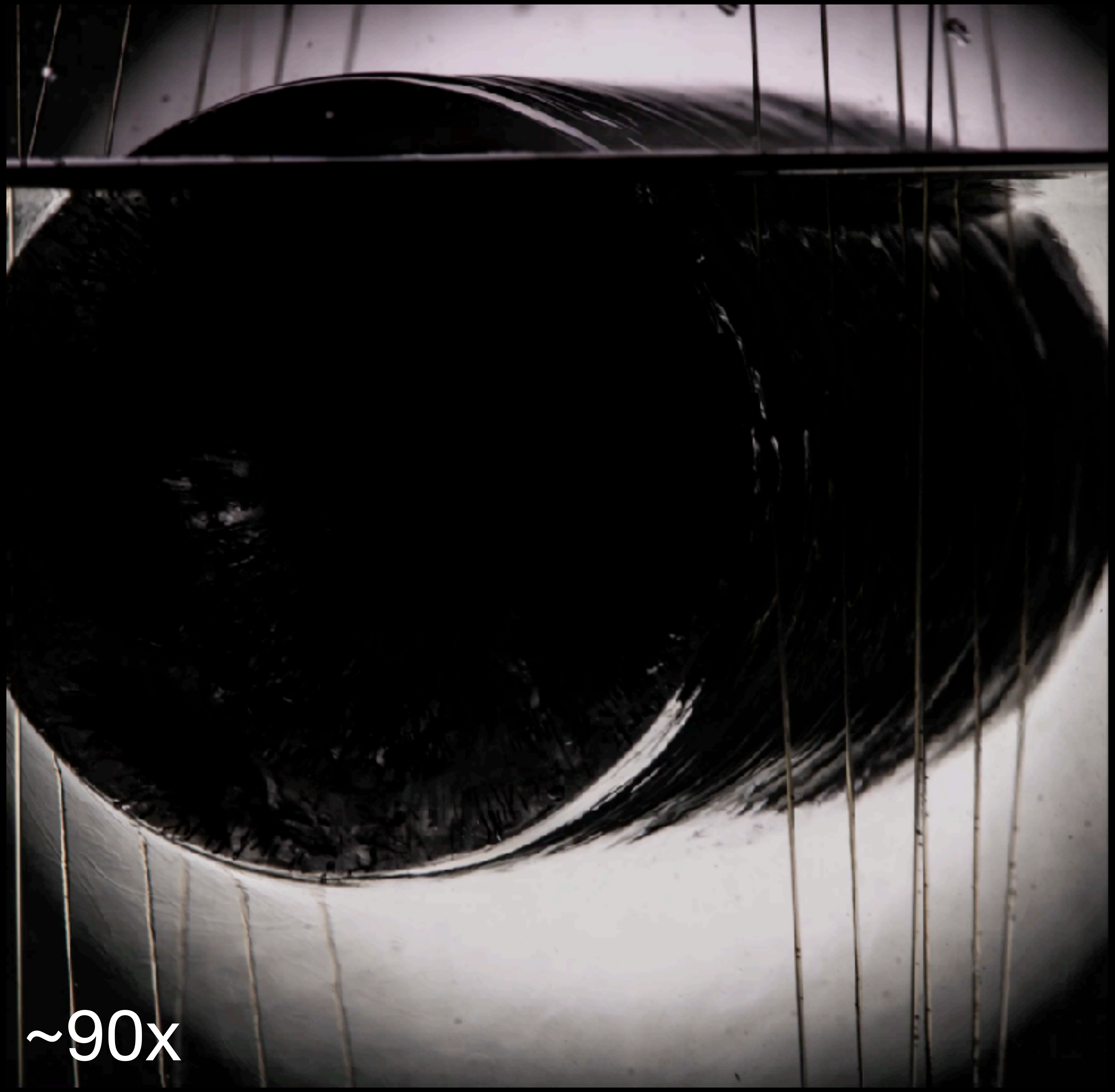


Morphology

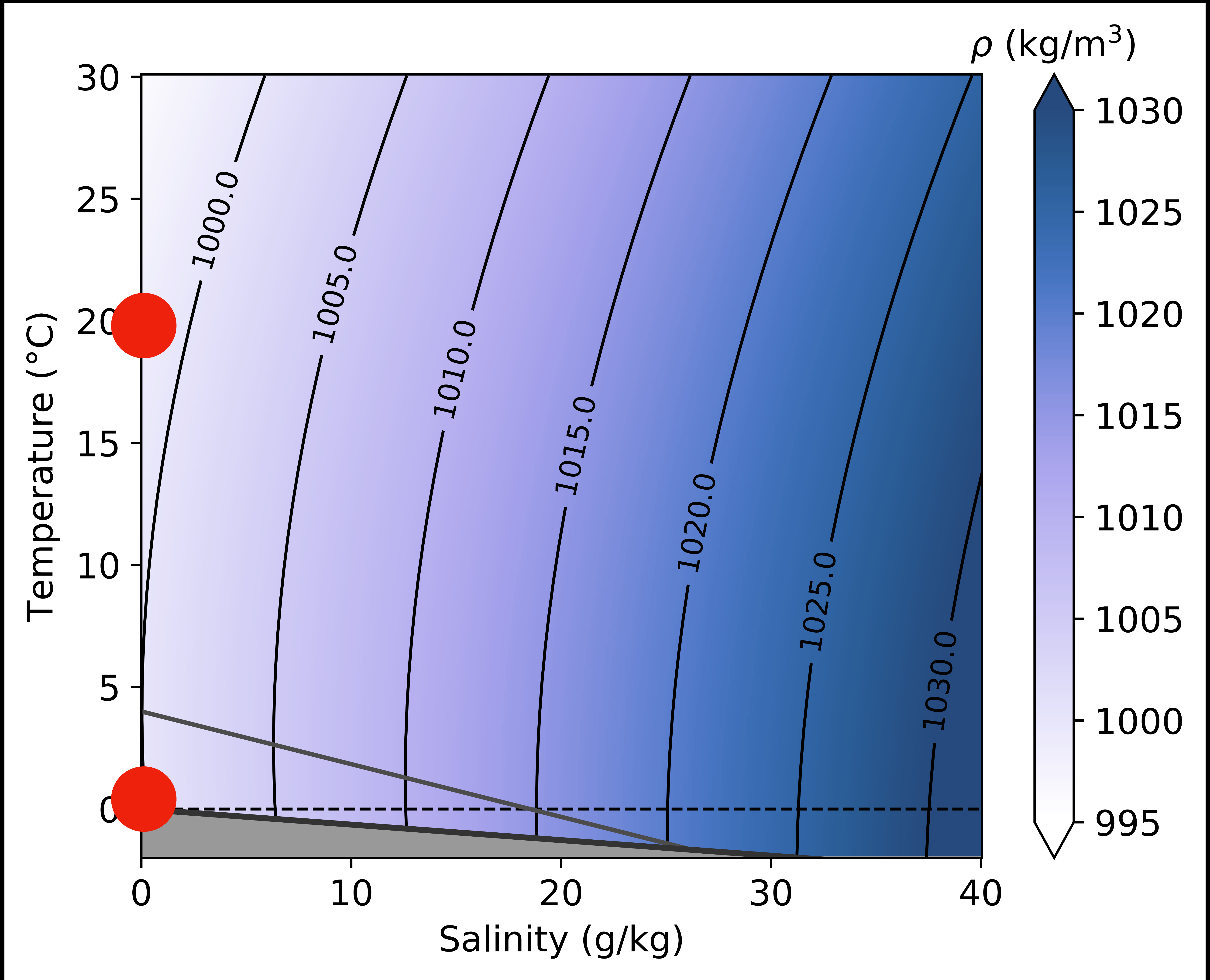
Salinity effect on morphology

Freshwater VS Salty water



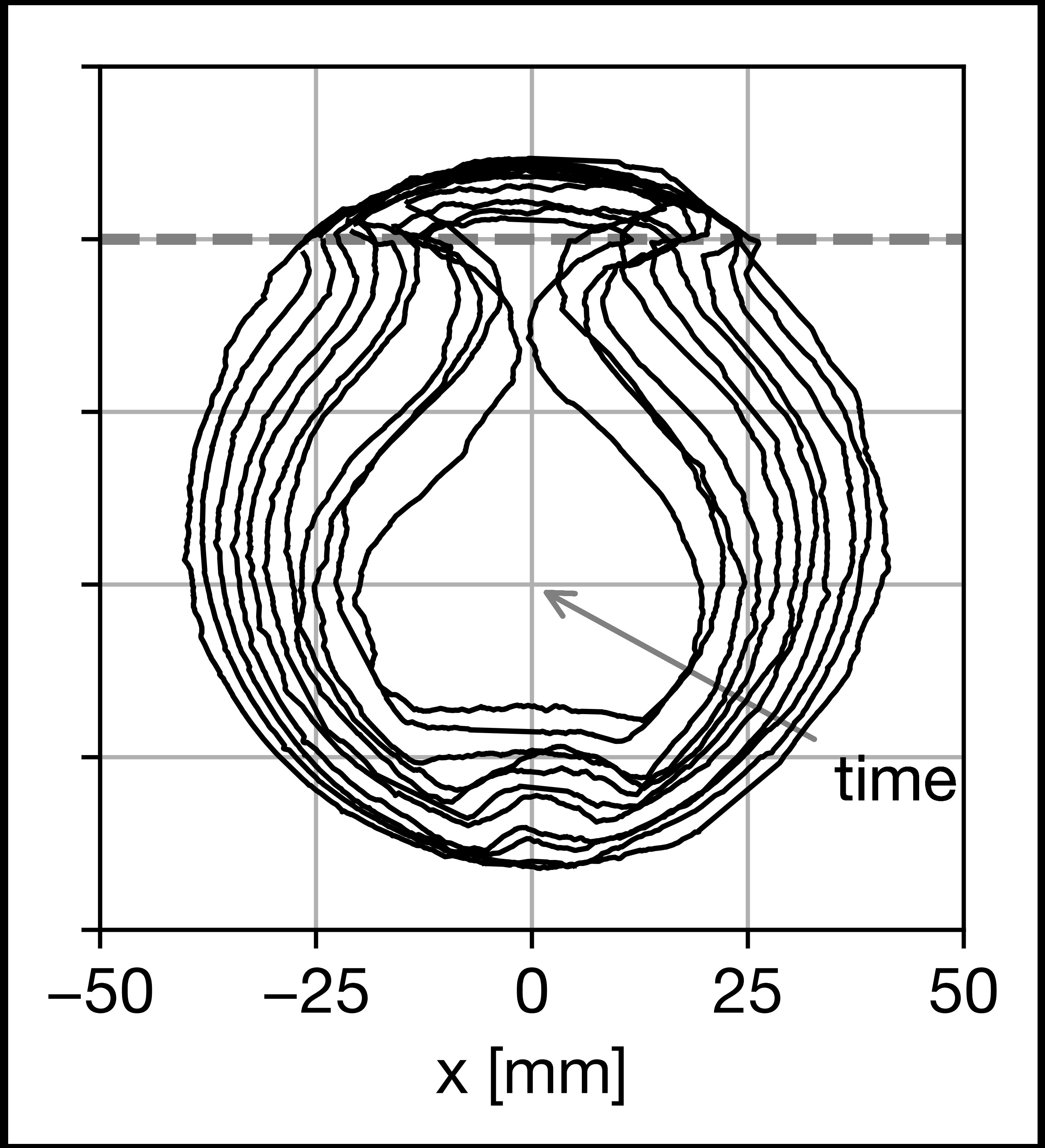


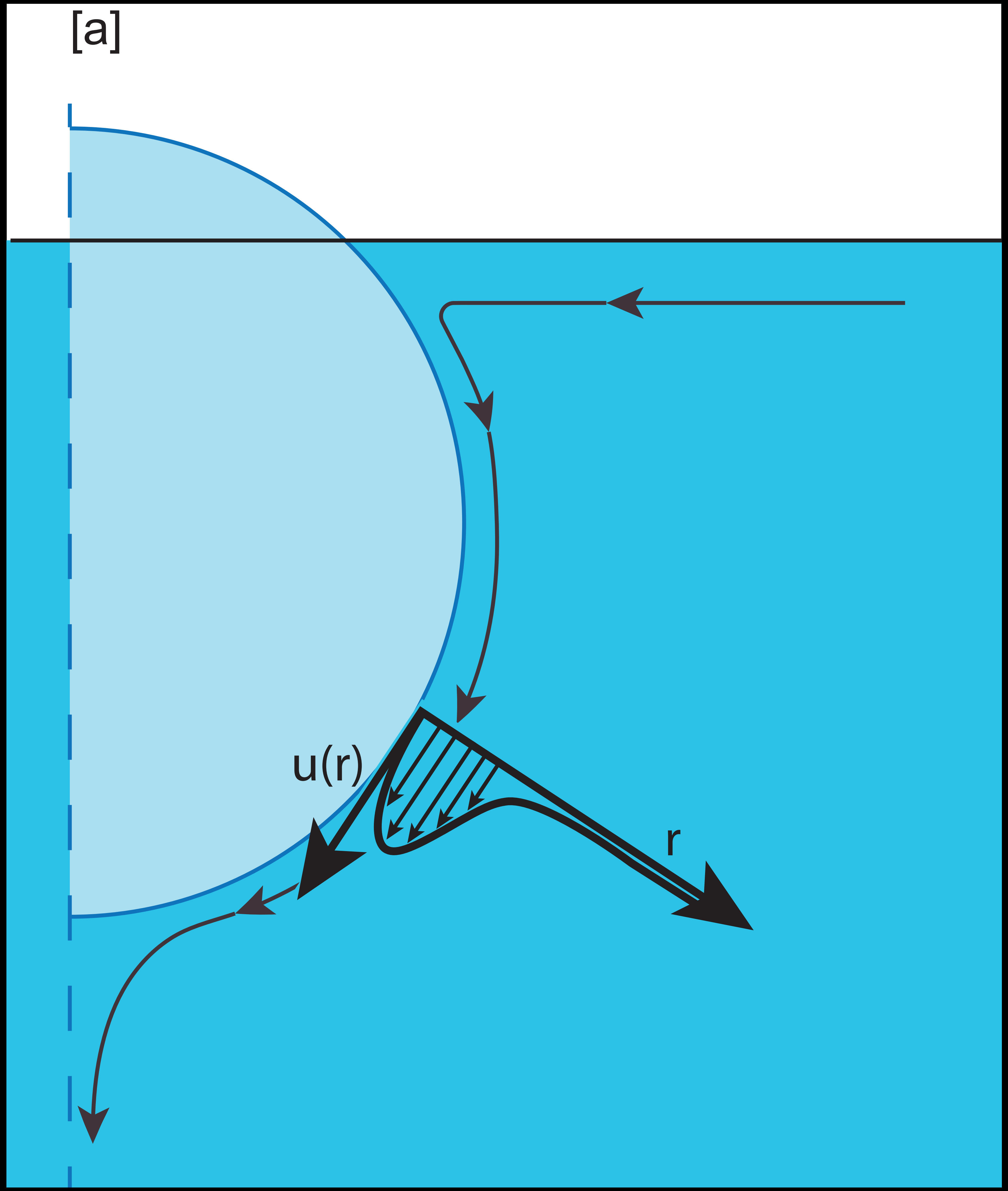
Density of fresh water



Cylinder in fresh water

Rotation prevented

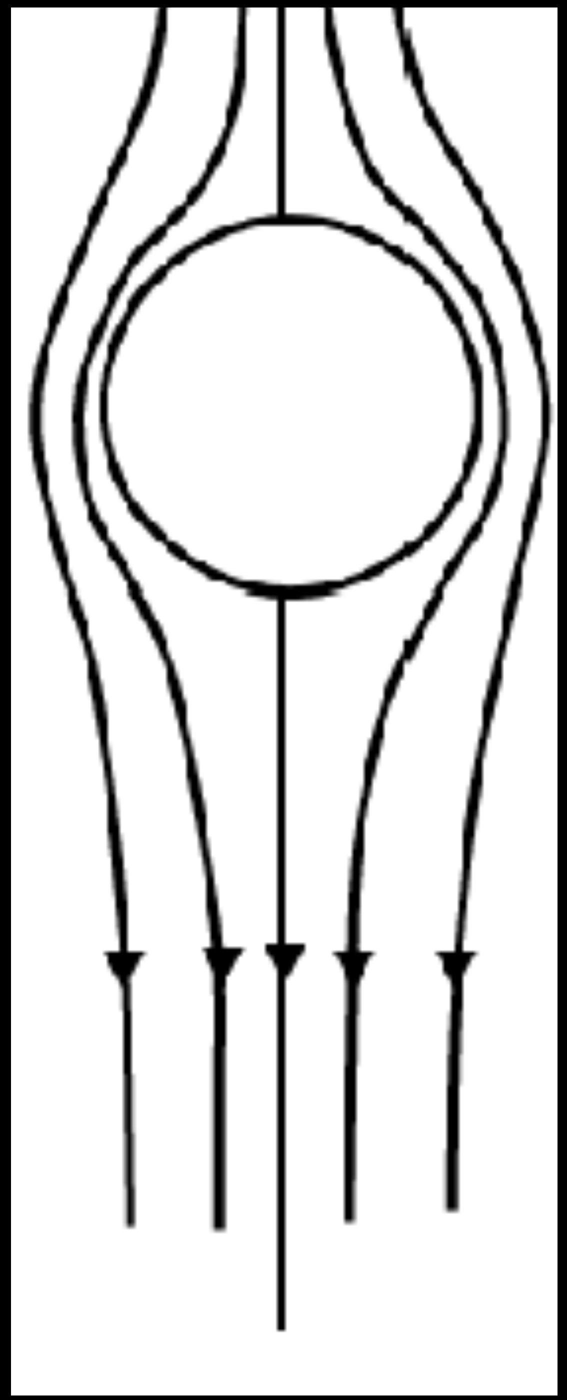




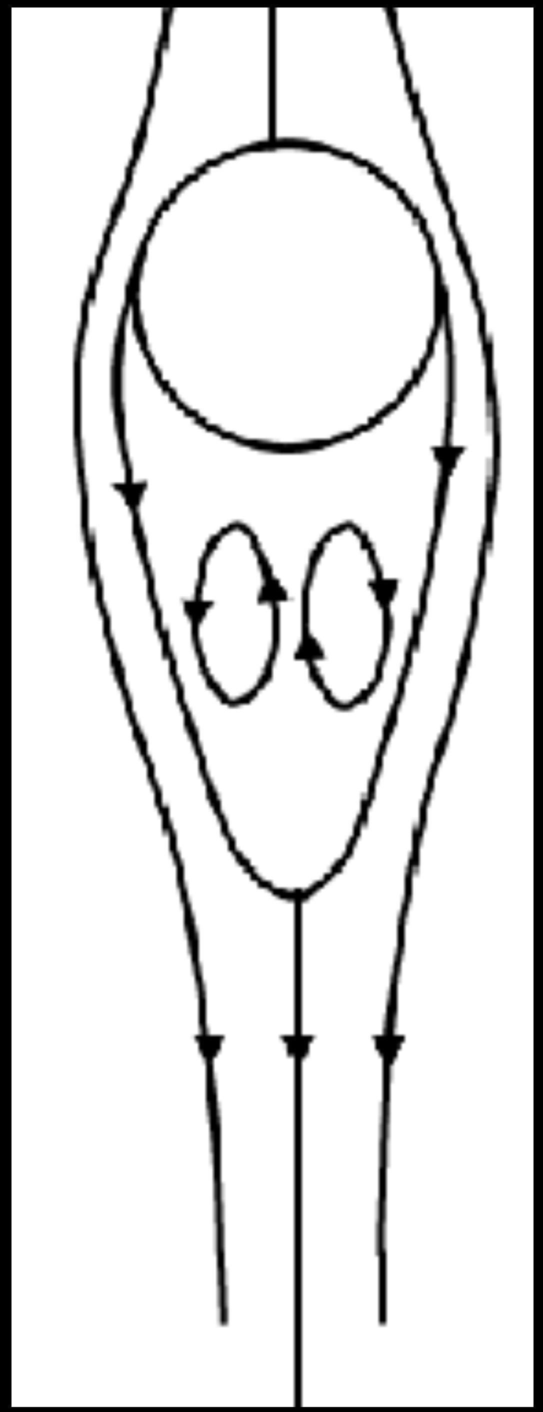
Flow past a cylinder

-> Re (Ra)

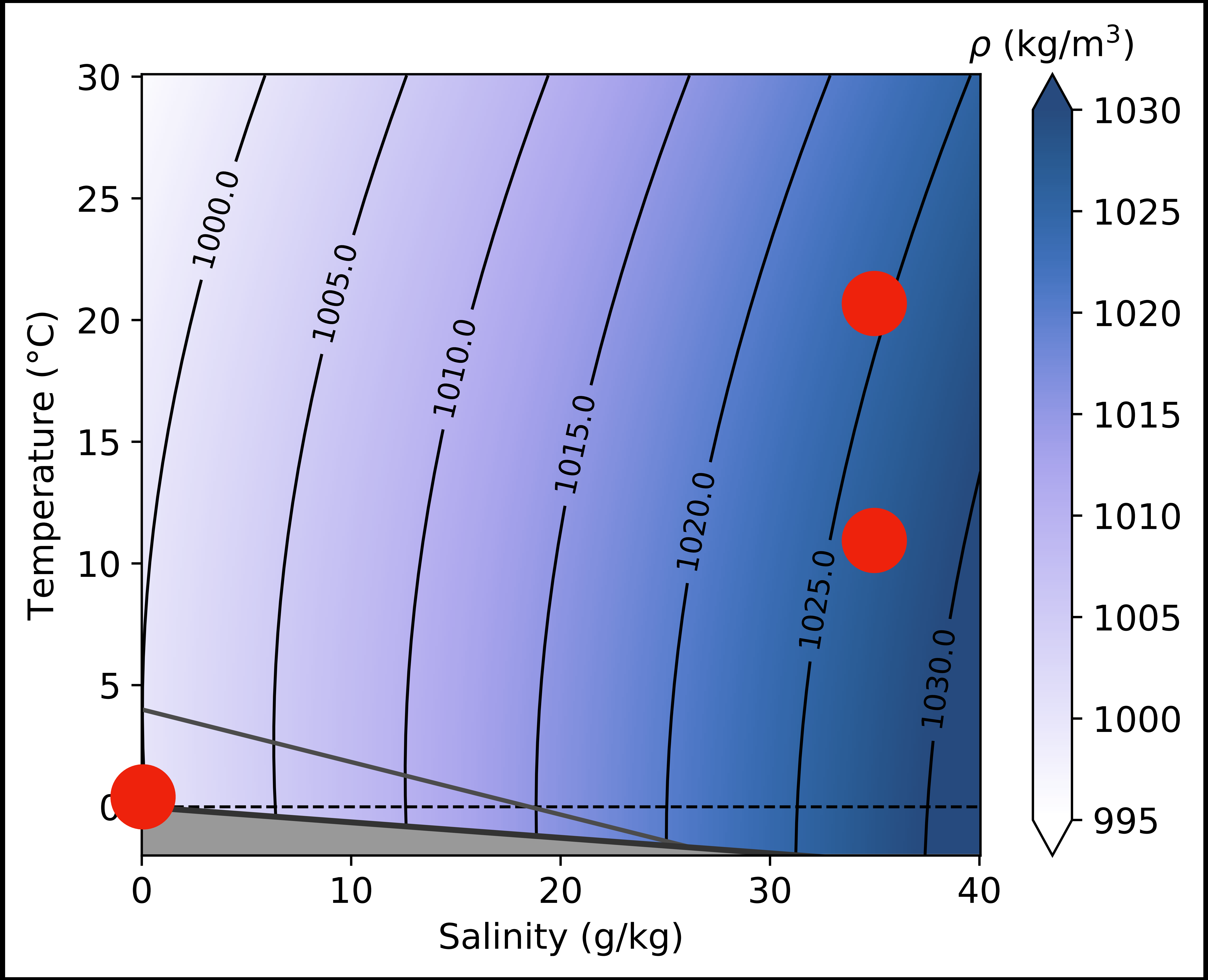
Low

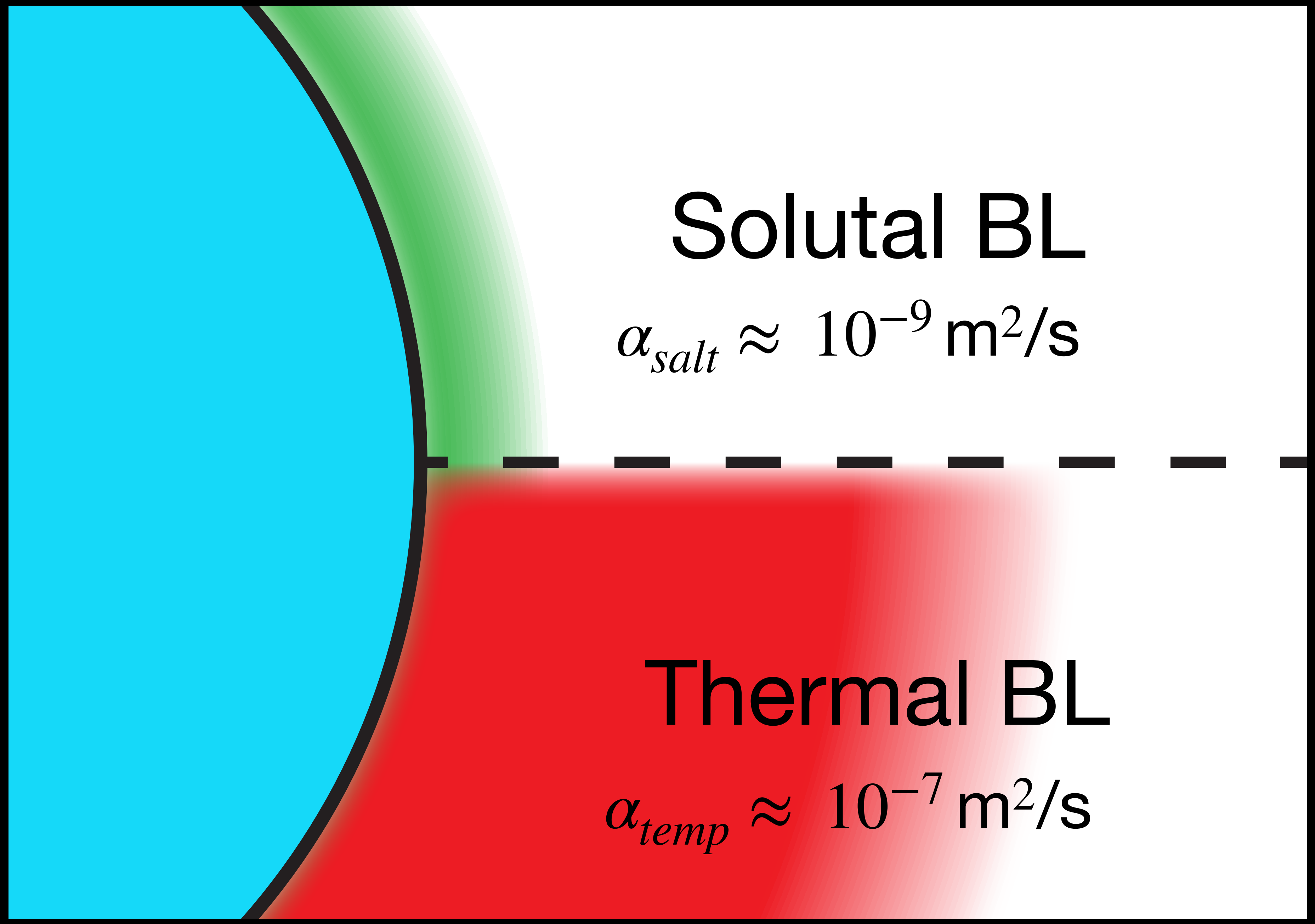


High

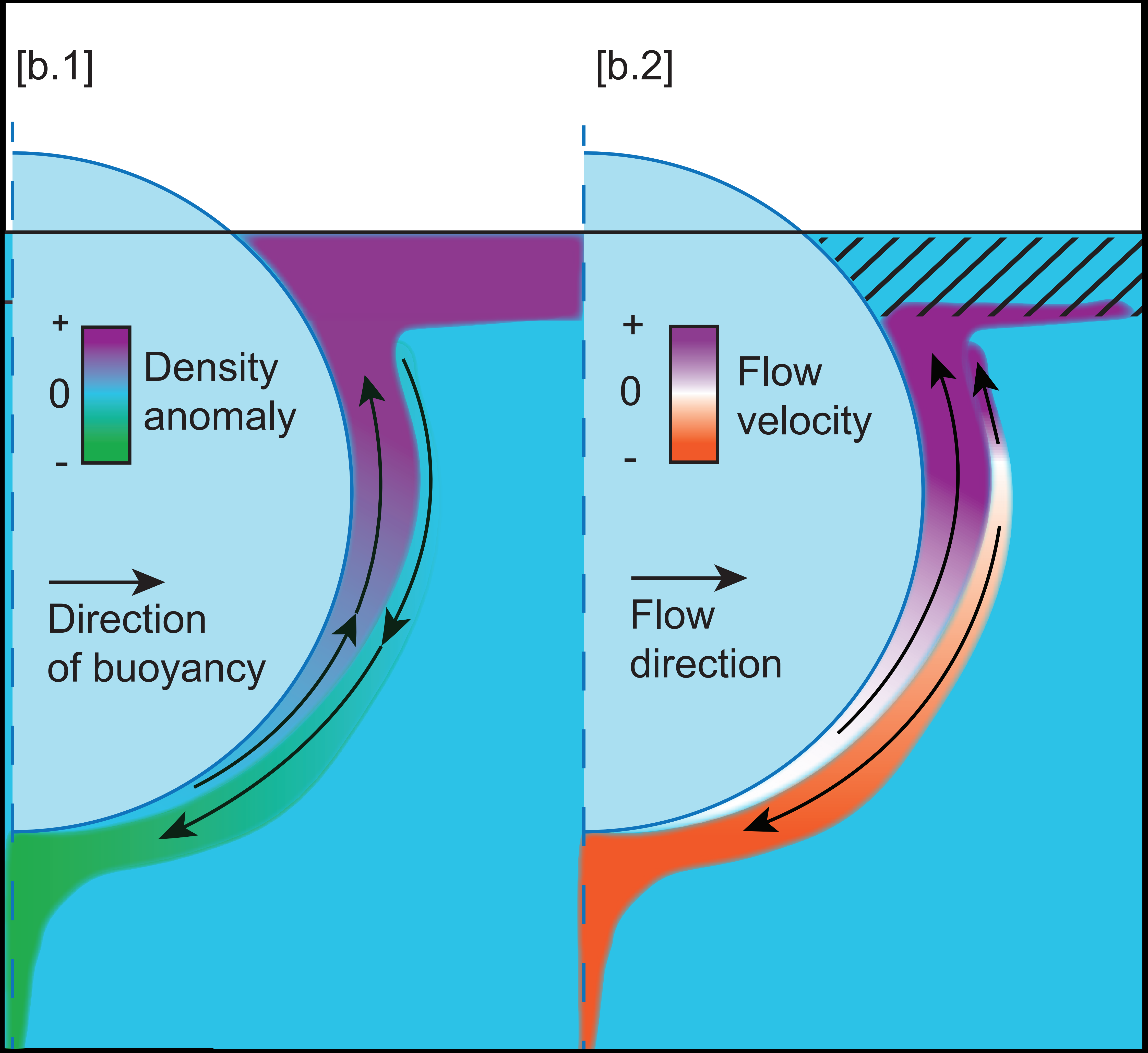


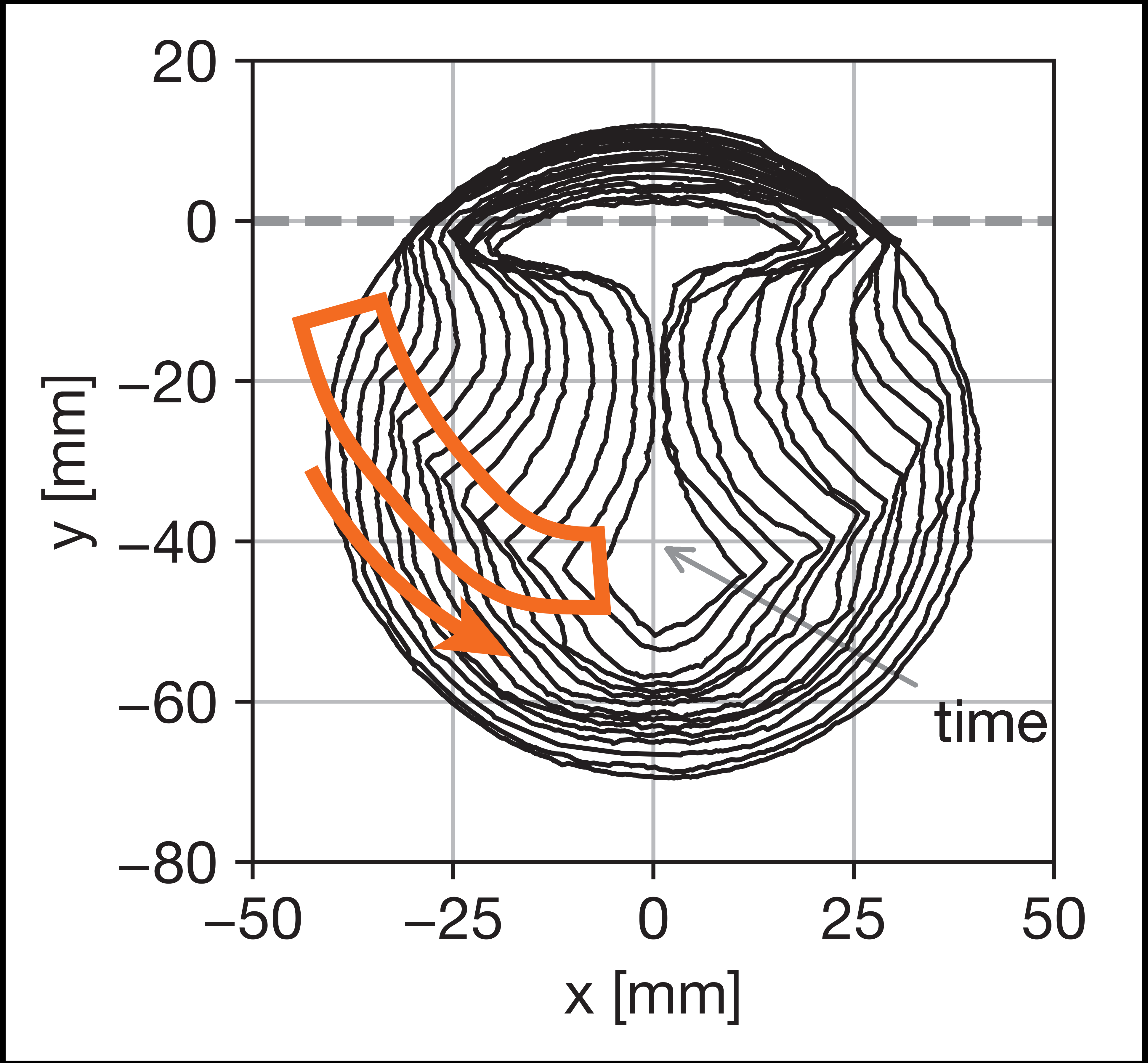
Density of salty water



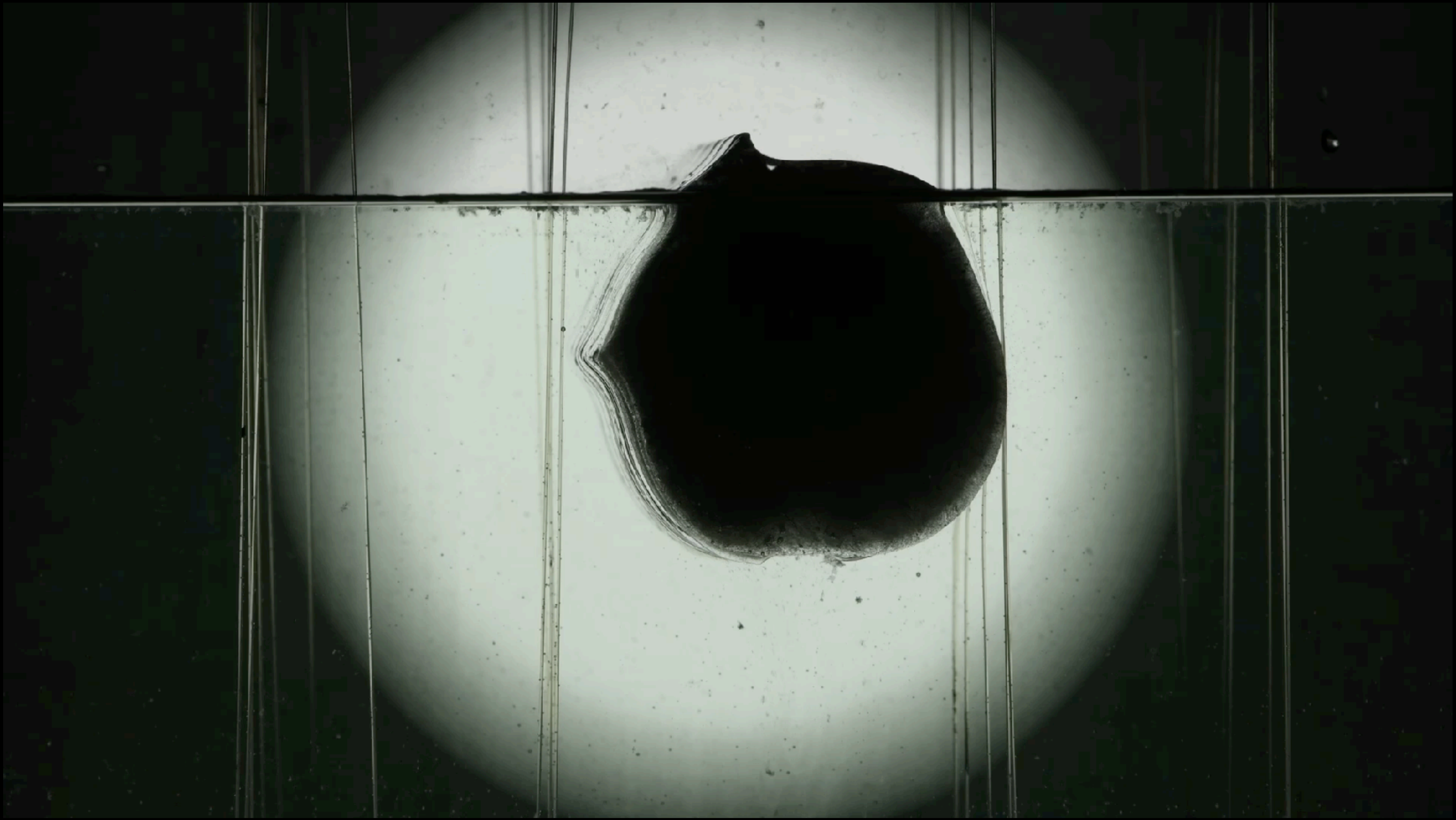


Salinity effect on morphology



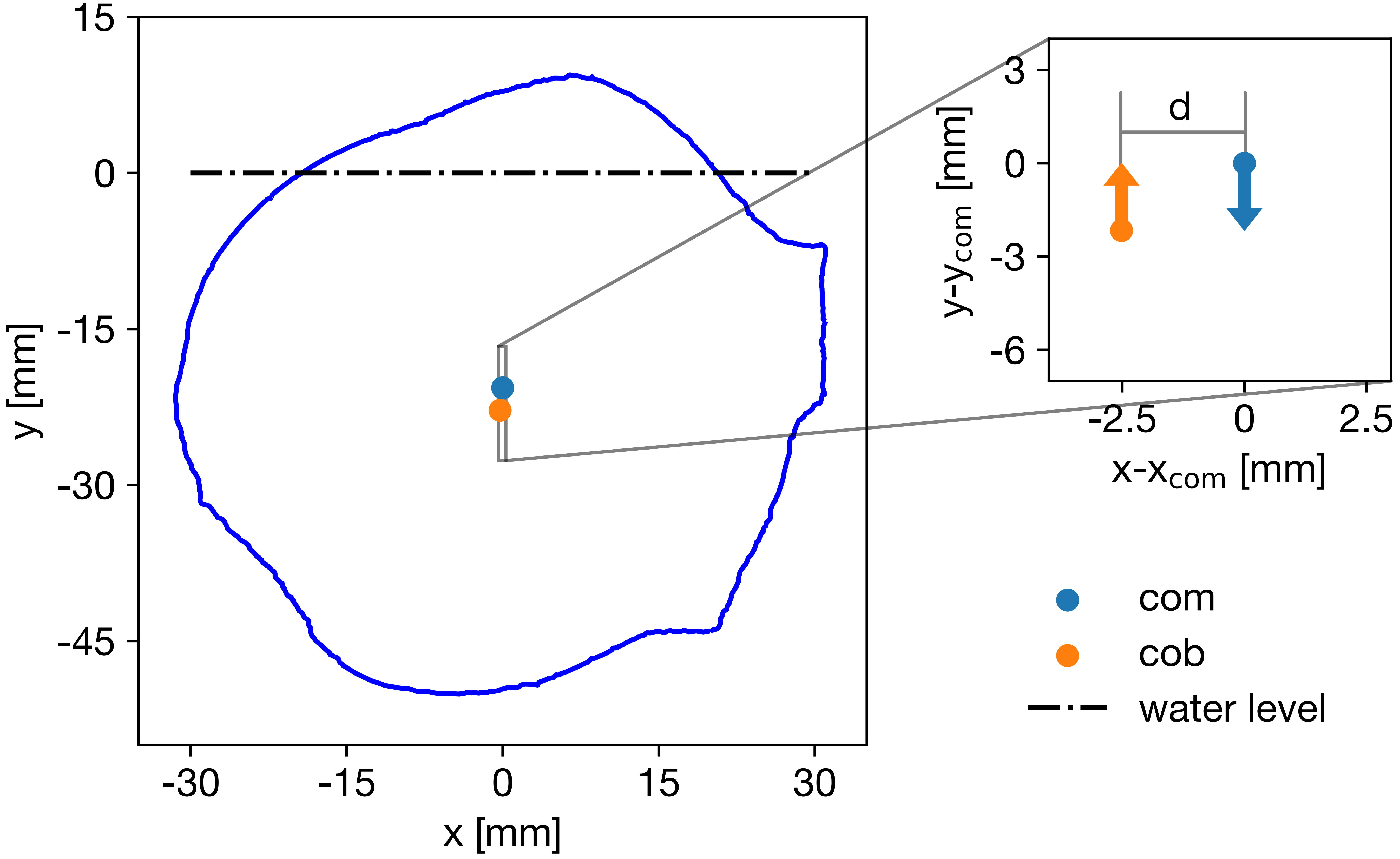


Rotations and Stability



A simple harmonic model

Forces on a floating cylinder



Newton's second law

$$\hat{e}_z : \; I\ddot{\theta} = \sum_i \tau_i = \tau_{buoy} + \tau_{drag}$$

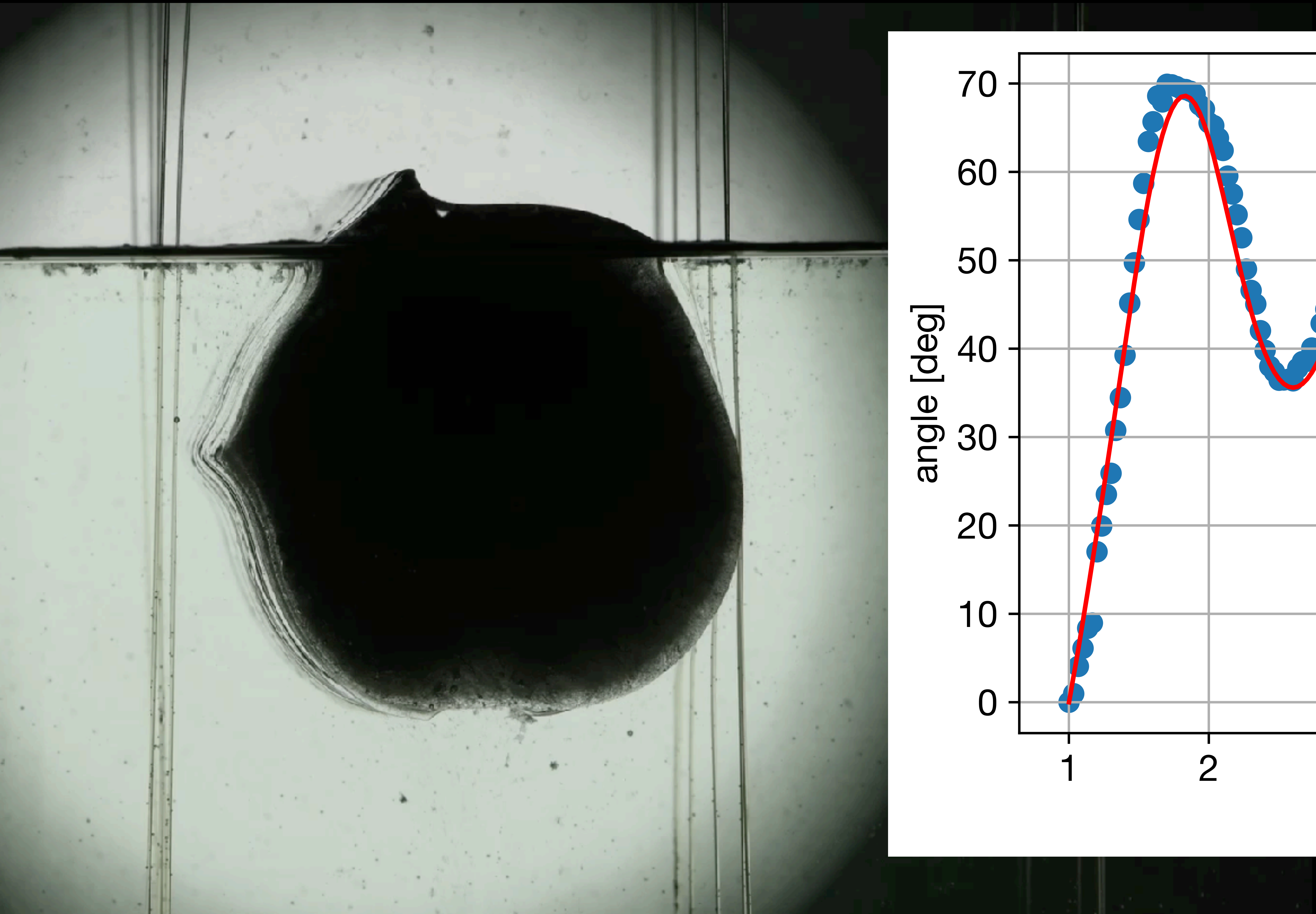
That is

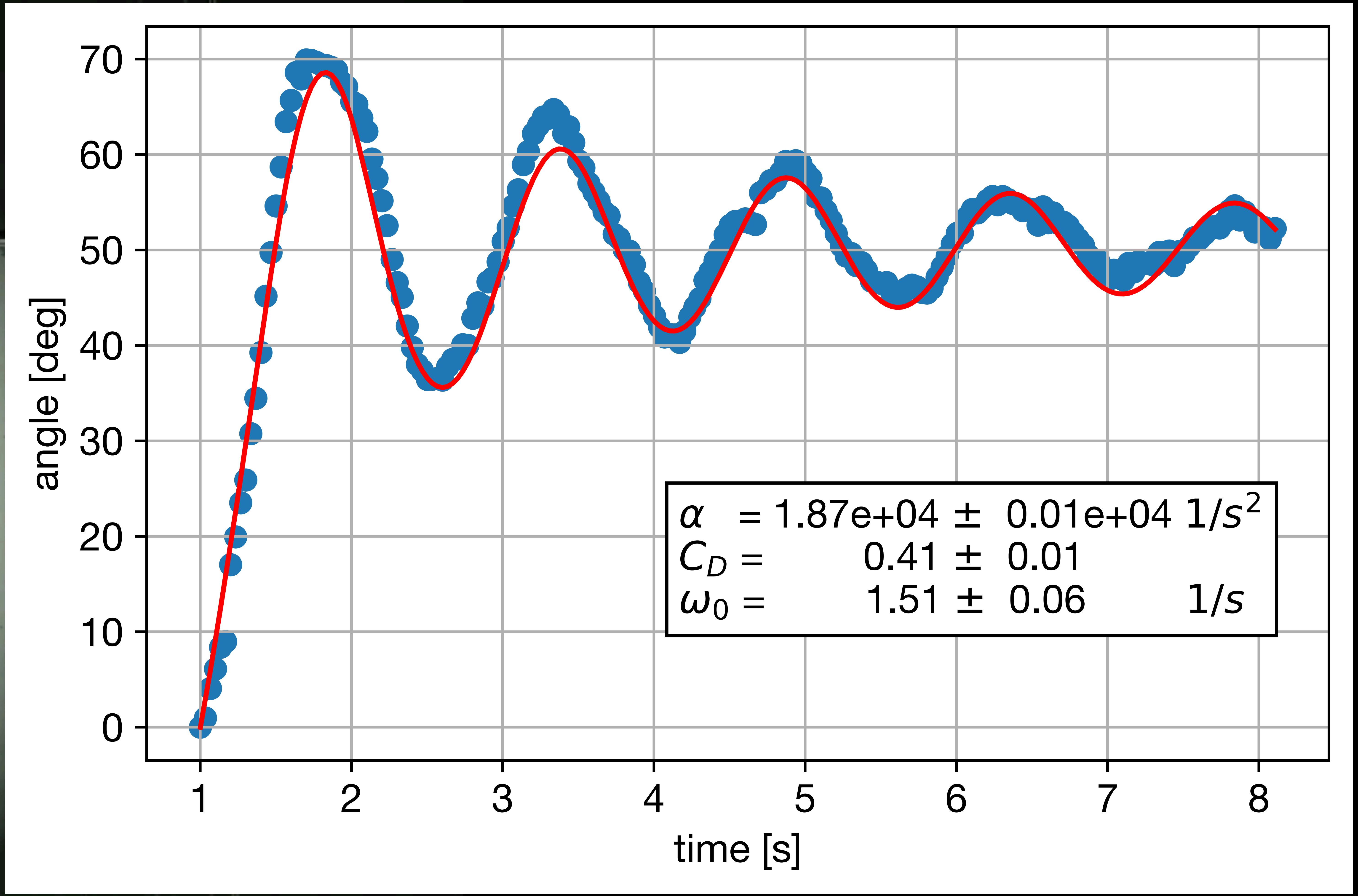
$$\frac{1}{2}\,\frac{\rho_i}{\rho_w}\,V\,r^2\,\ddot{\theta} = -\,g\,d(\theta)\,\frac{\rho_i}{\rho_w}\,V - \frac{1}{2}\,A\,C_D\,r\,\dot{\theta}\,|\dot{\theta}|$$

$$\ddot{\theta} = -\,\alpha\,d(\theta) - 2\frac{\rho_w}{\rho_i}\,C_d\,\dot{\theta}\,|\dot{\theta}|$$

A simple harmonic model

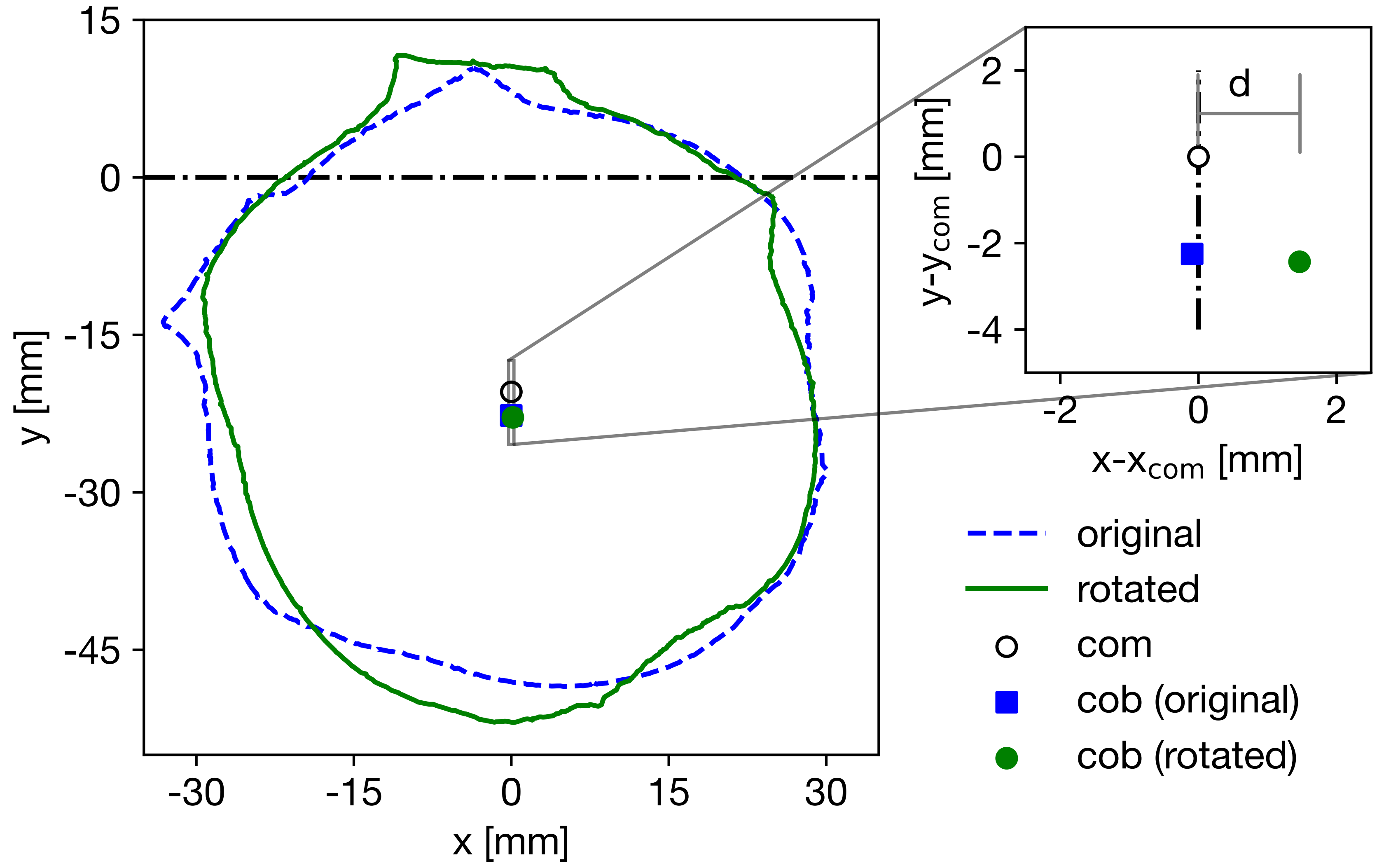
Fit to experimental data





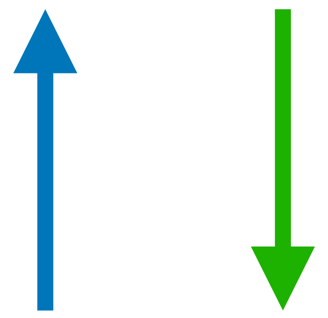
Stability of cylinders in different conditions

Effect of time and salinity

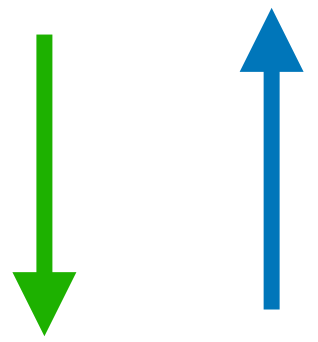


$$d = x_{com} - x_{cob}$$

$$d > 0$$



$$d < 0$$



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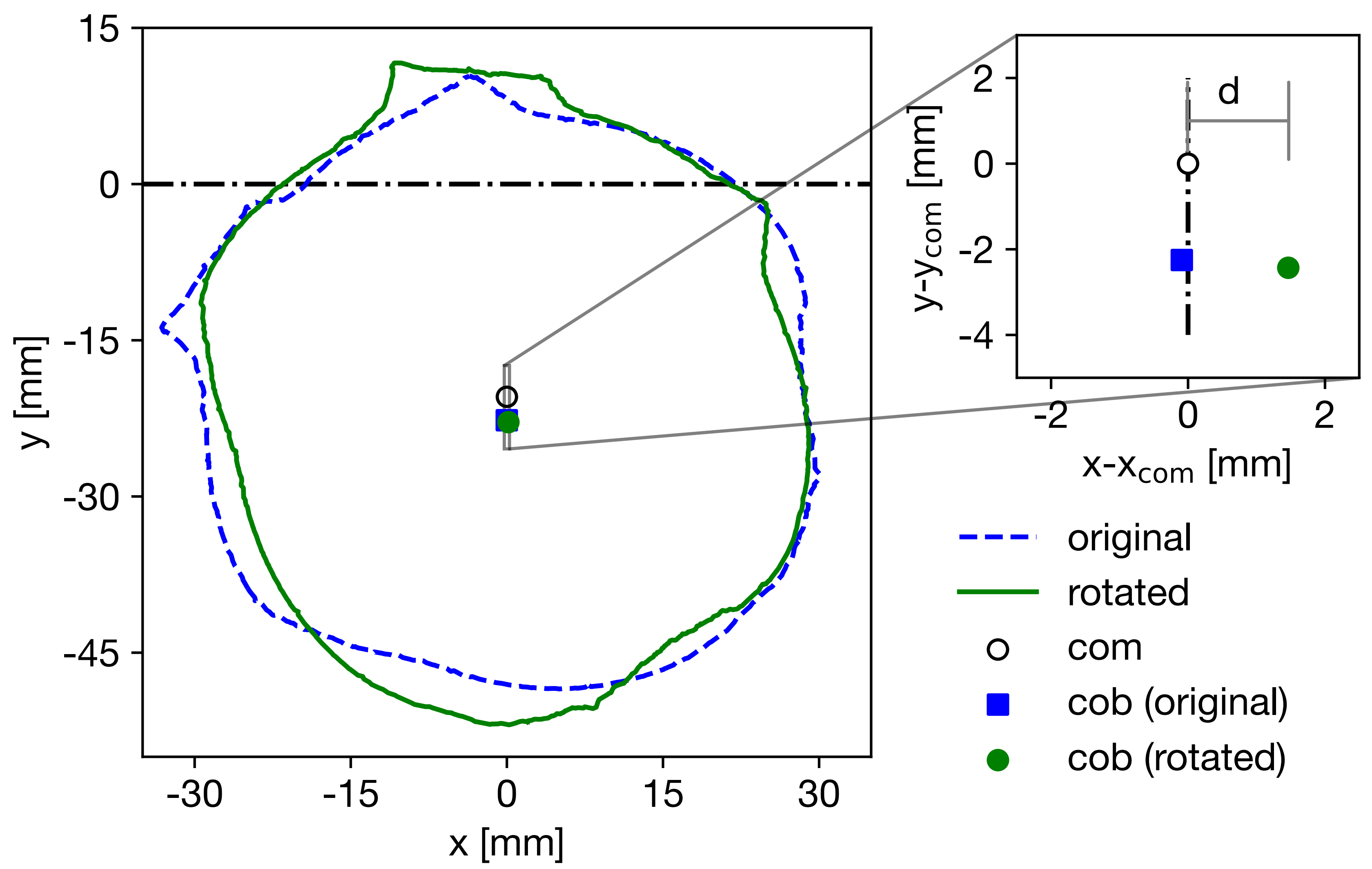
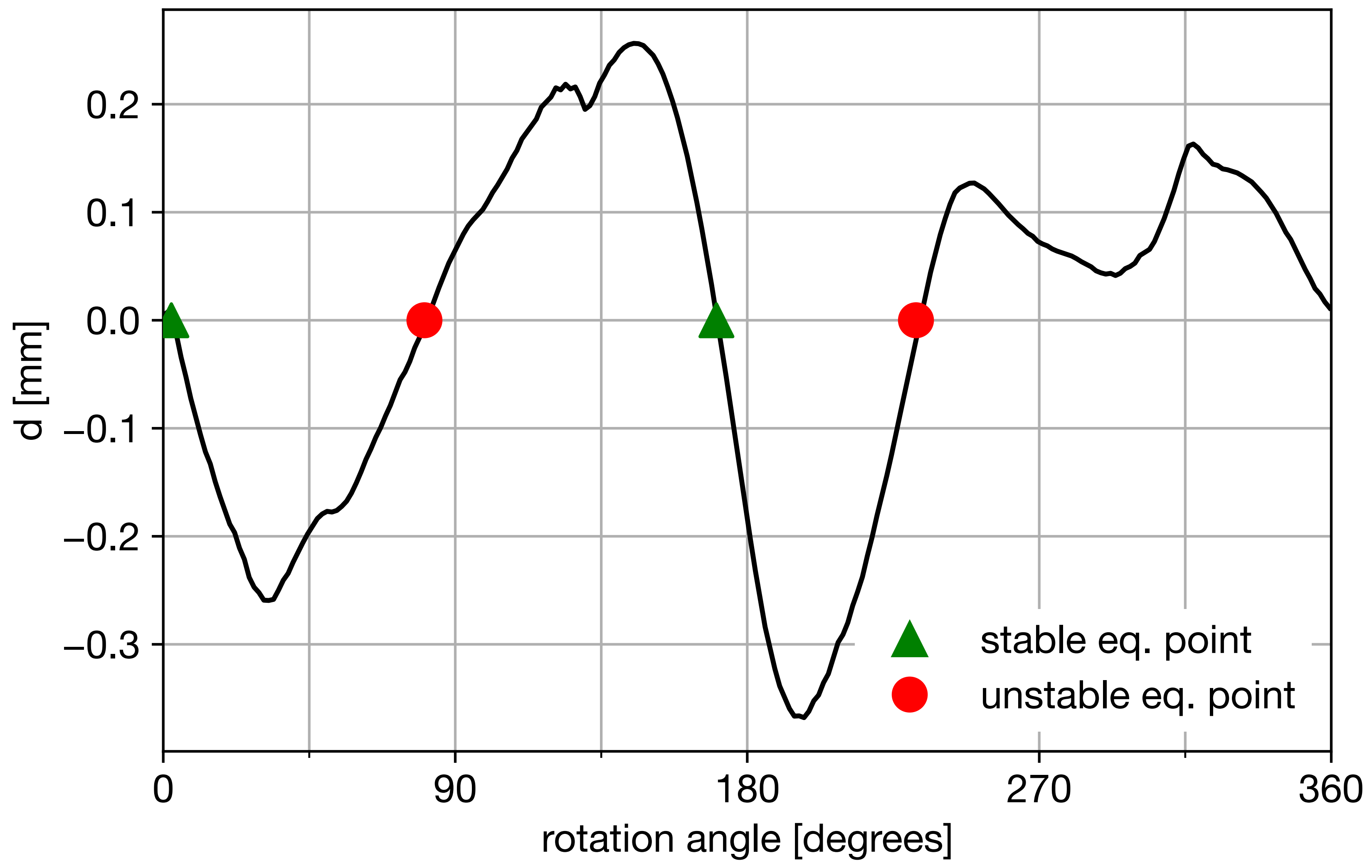
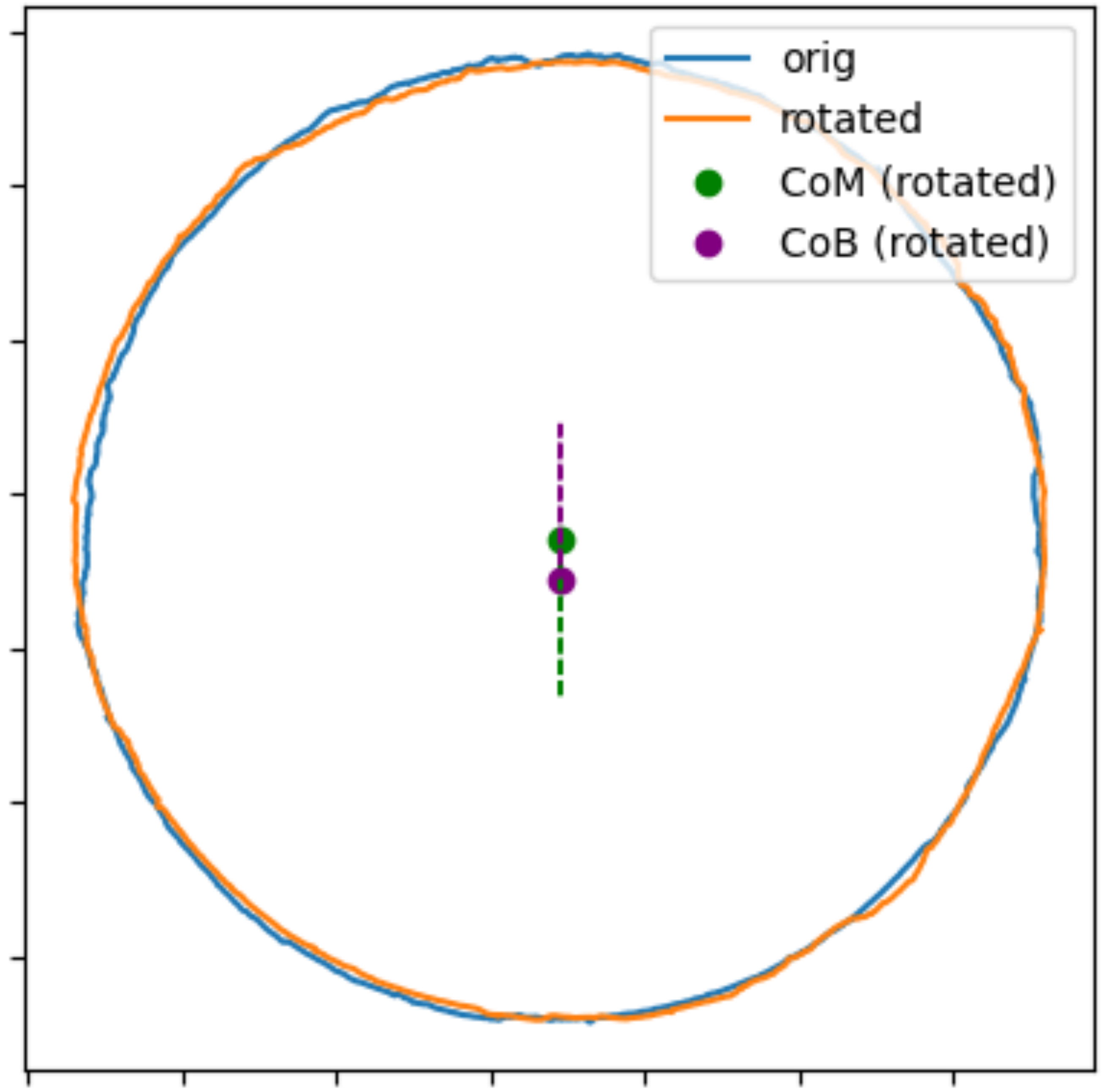
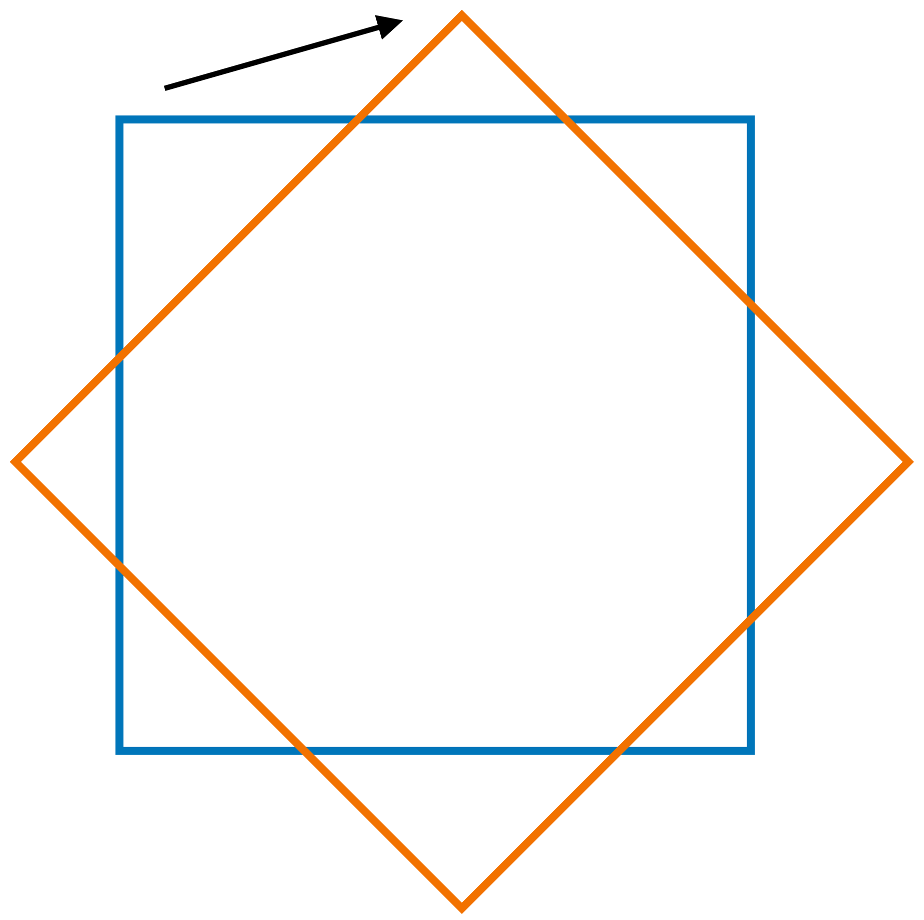






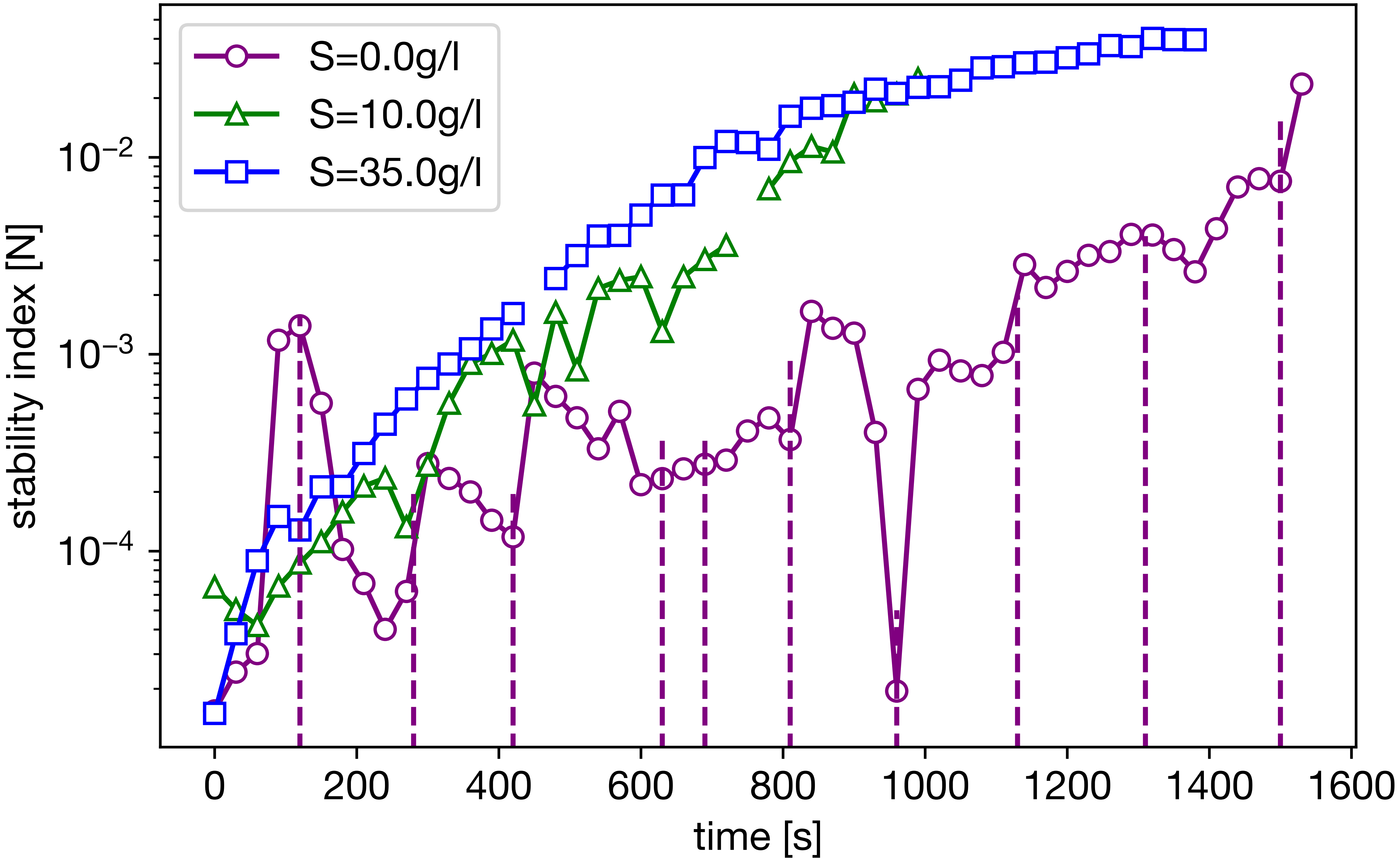


$$\text{S.I.} = (\theta_{unst} - \theta_{st}) \cdot \max(d(\theta)) \cdot g\rho^* S$$

$$[\text{S.I.}] = N$$

Stability of cylinders in different conditions

Effect of time and salinity

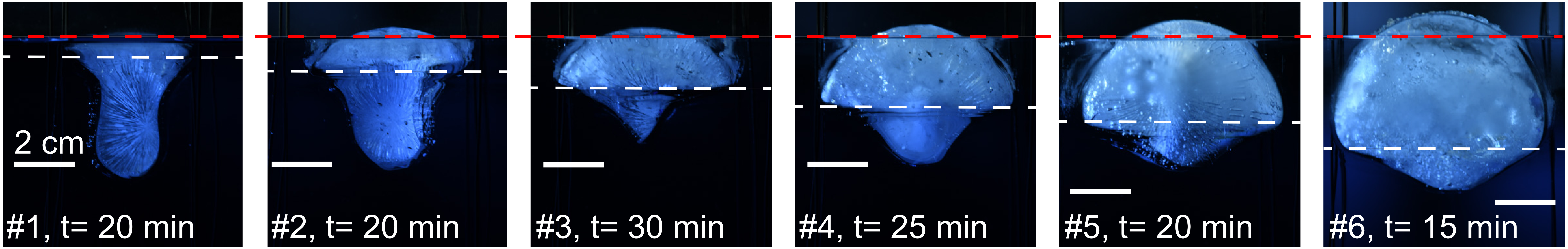


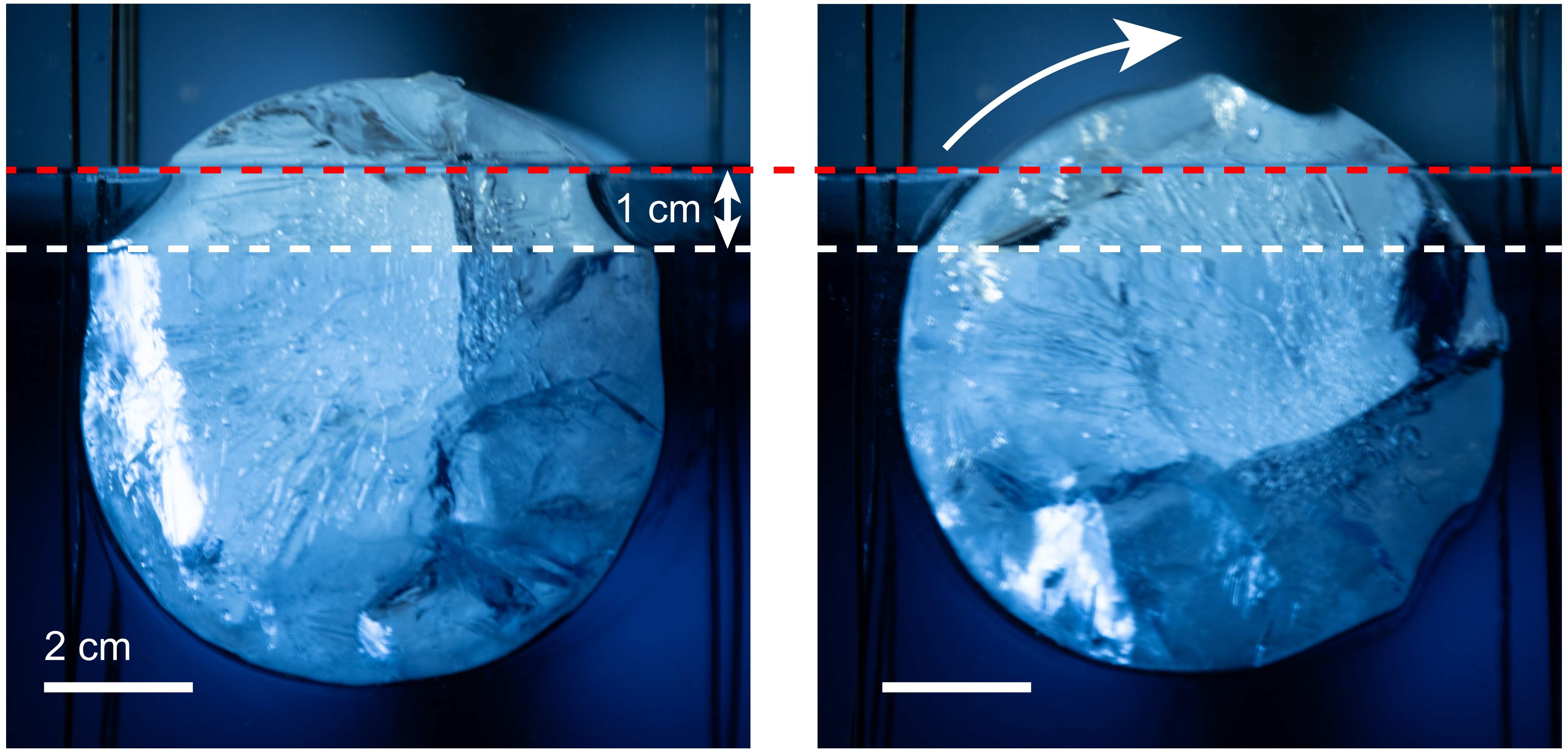
Rotations energetically favourable

Stability increasing in time in salty water

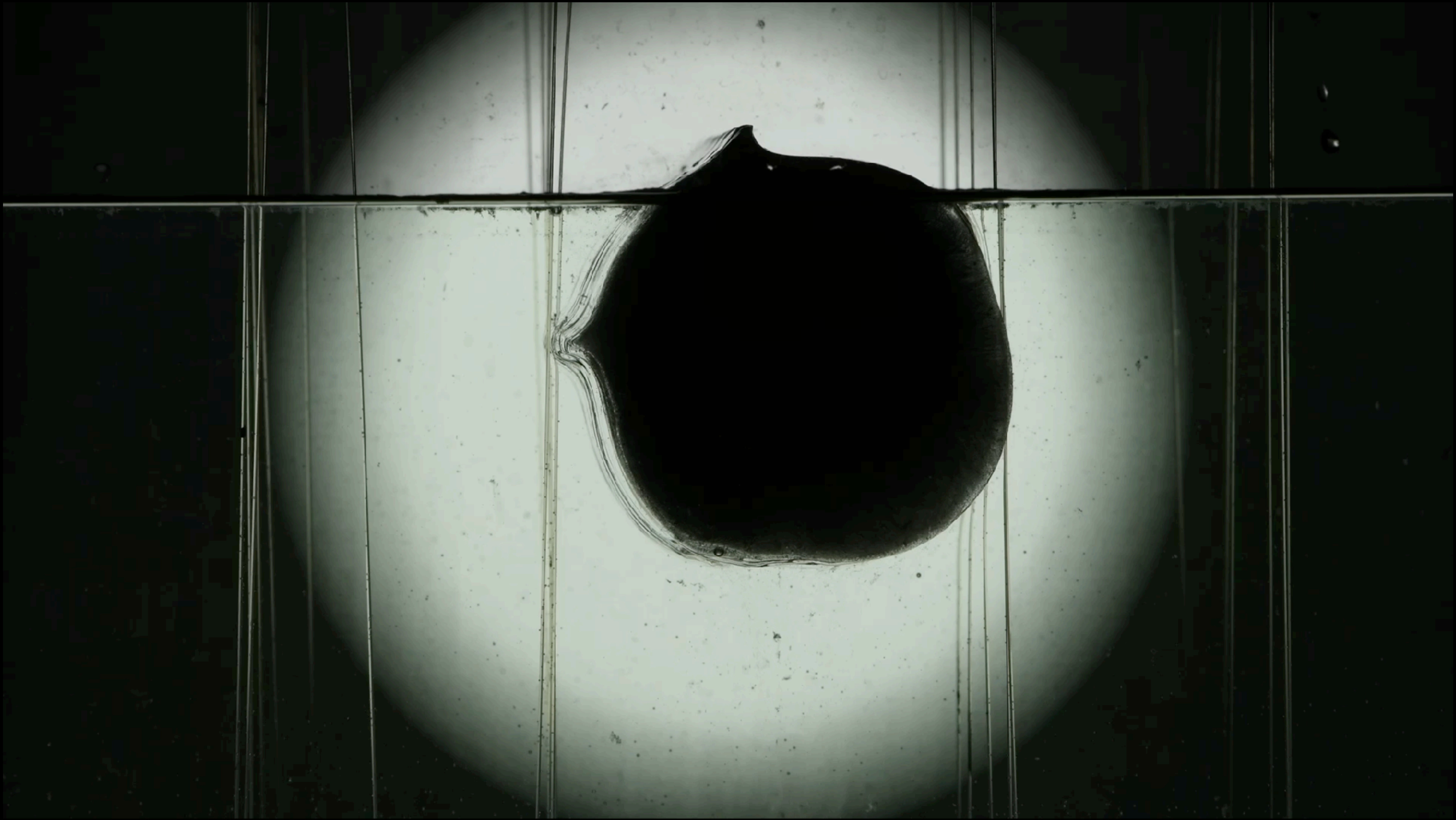
Stratified water

Stratified water



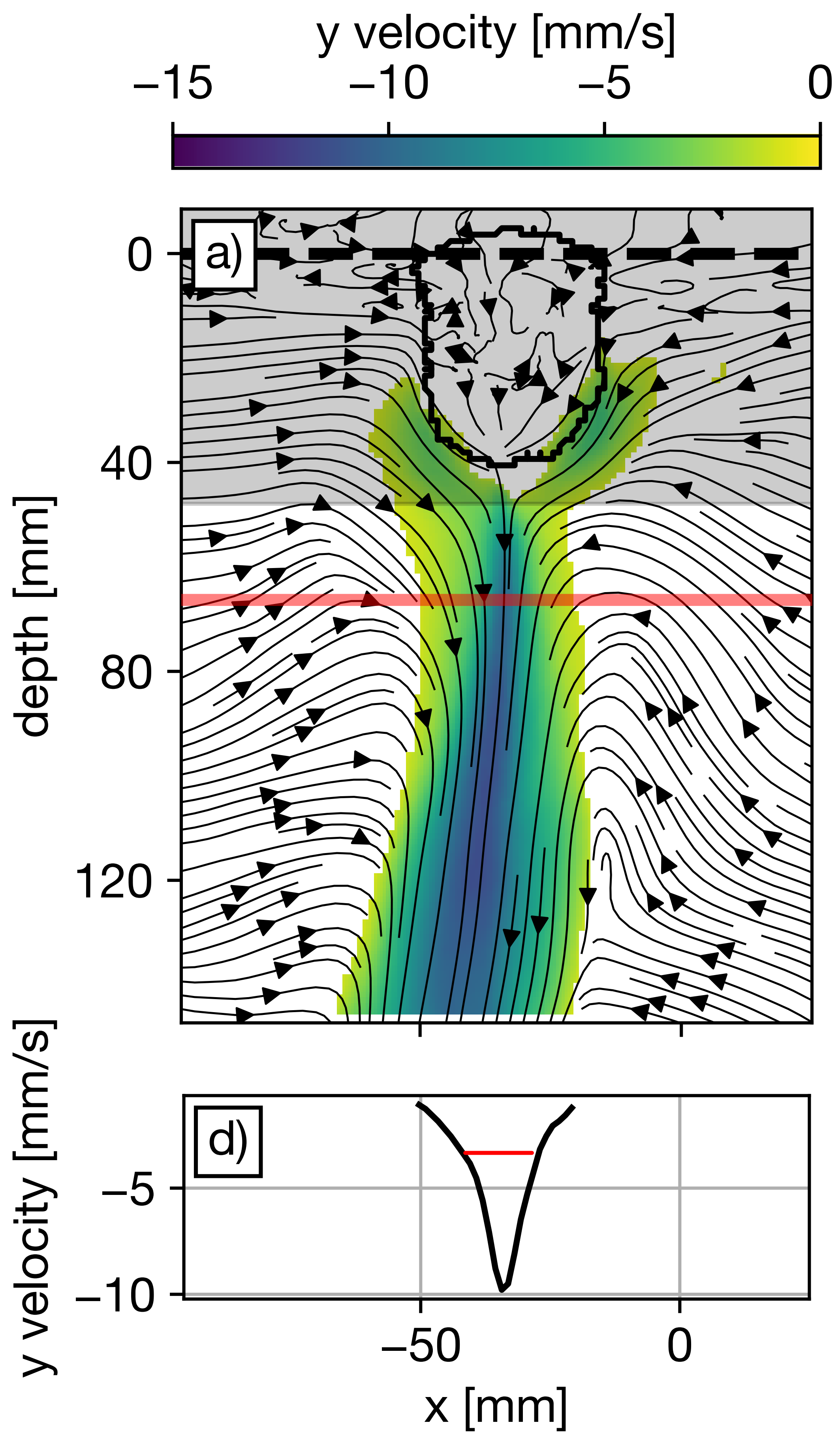


Plumes and heat transfer

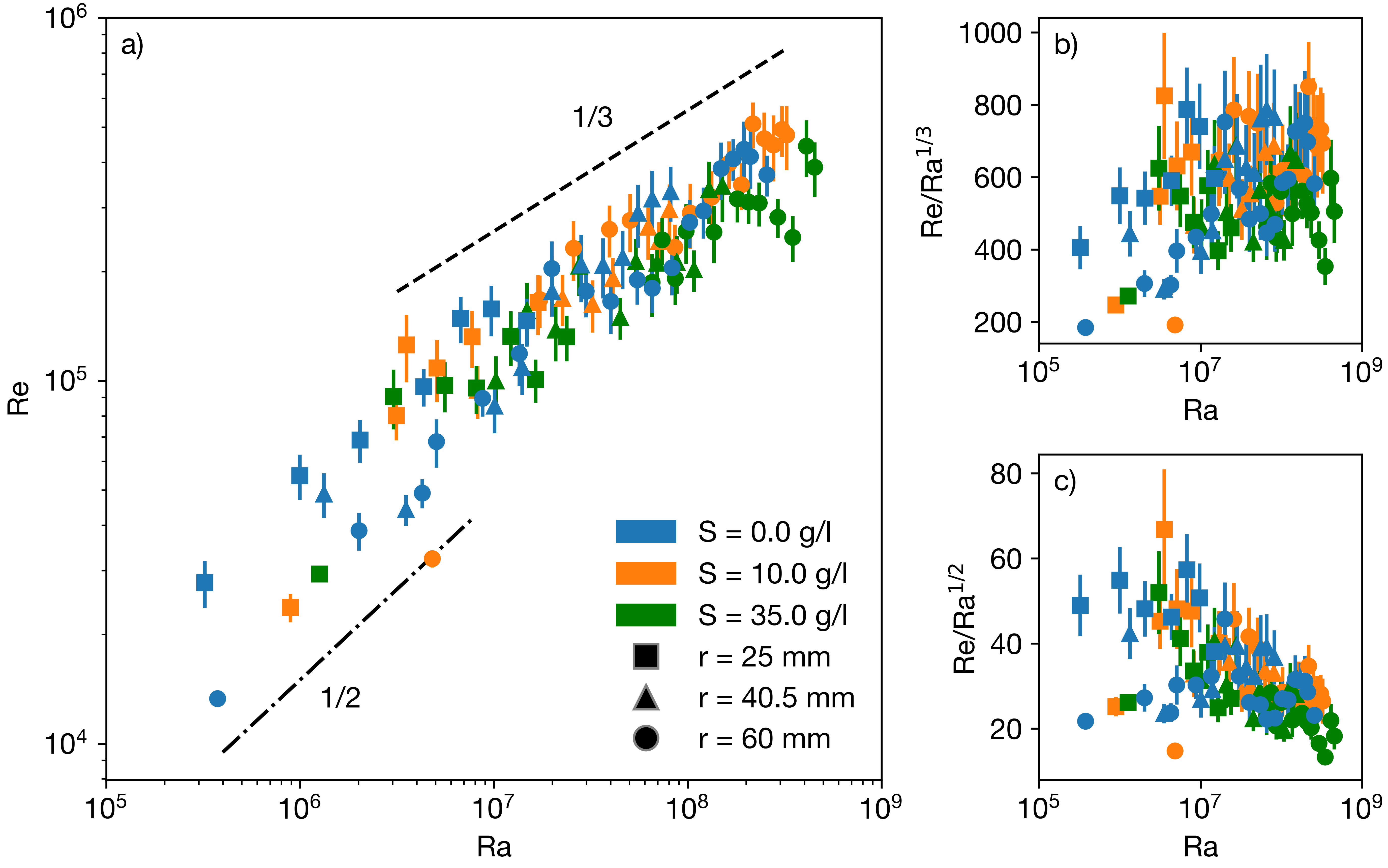


Sinking plume

Reynolds number

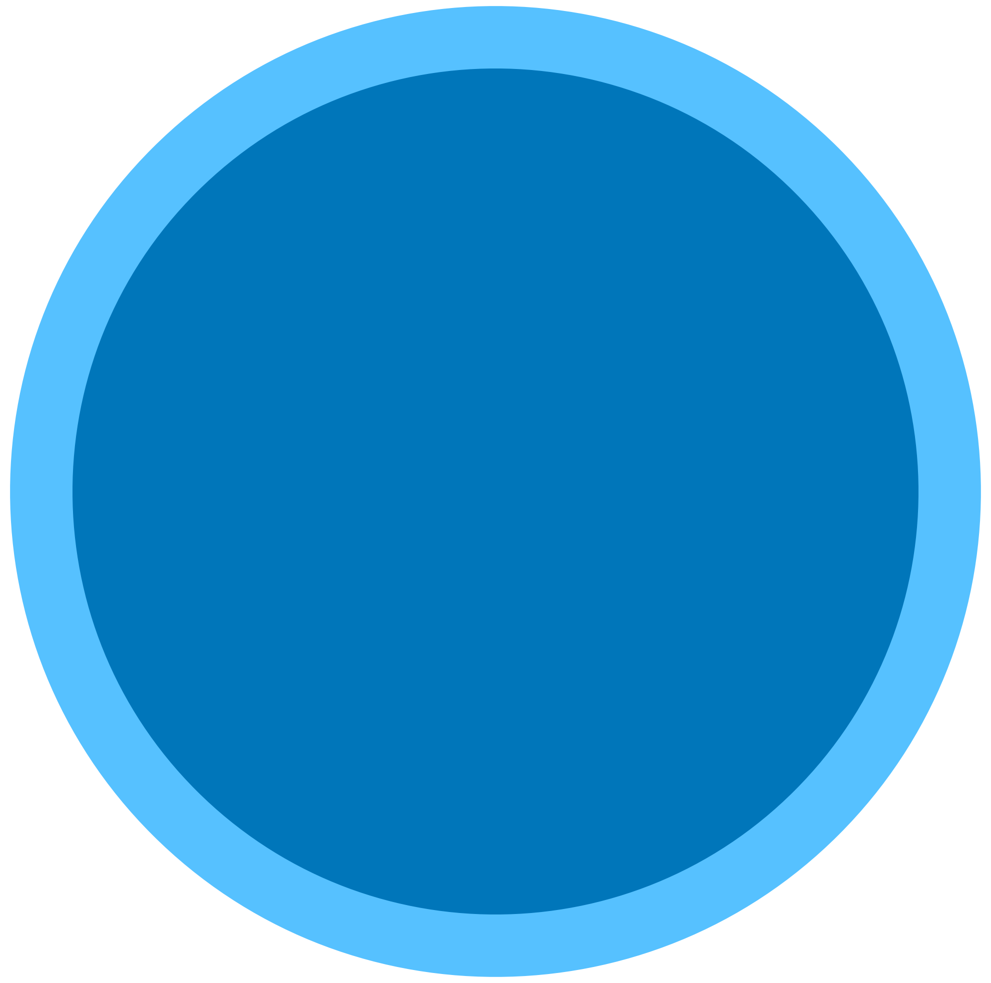


$$Re = \frac{u(2R)}{\nu}$$



Melting under convection

Nusselt number



$$\rho_{ice}\frac{\partial V}{\partial t}[L_f + c_s(T_{initial} - T_{melt})] = h(T_{water} - T_{melt})S_{lat} + \lambda_{th}\left\langle \frac{\partial T}{\partial \hat{n}} \right\rangle_S$$

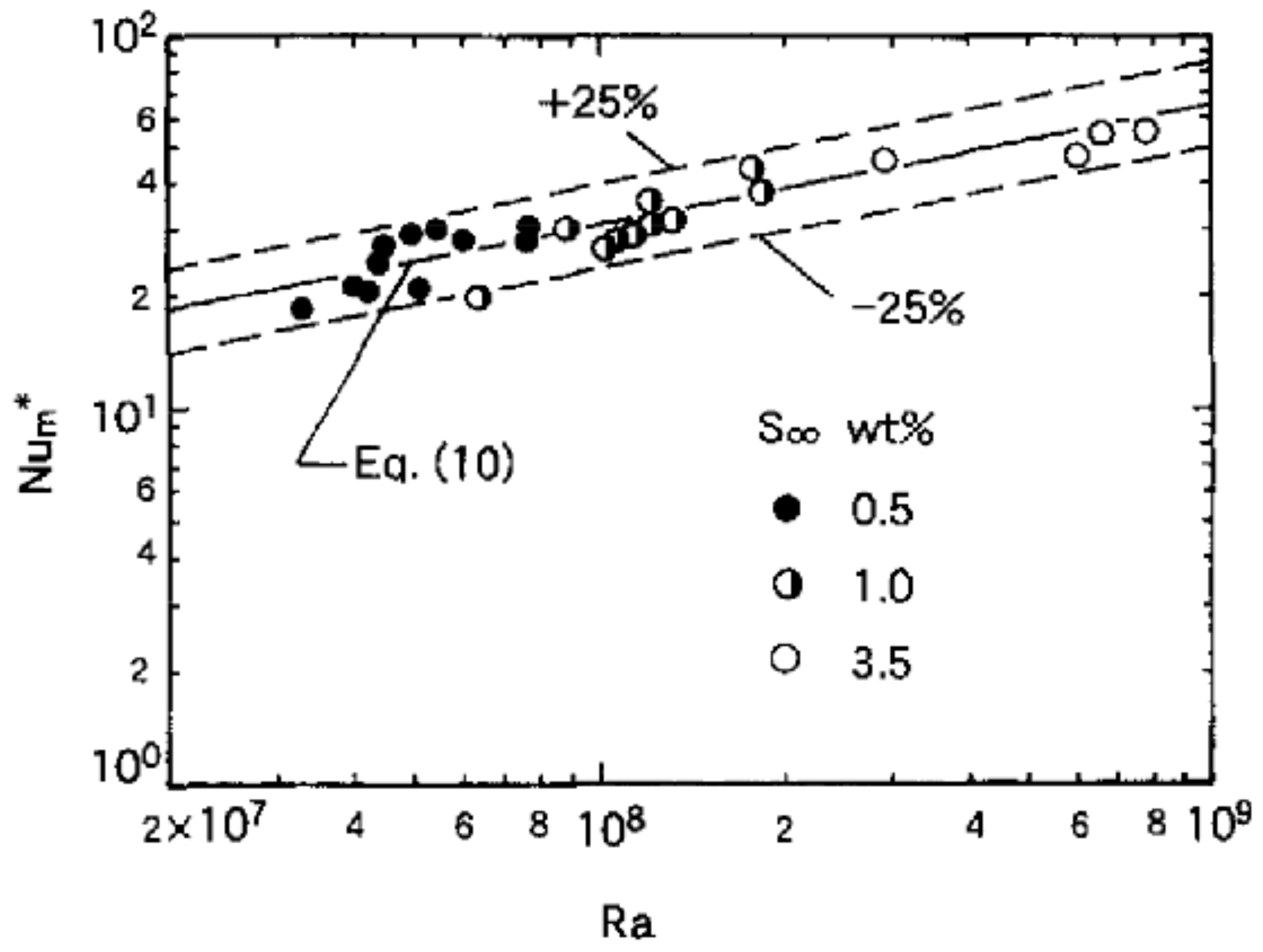
$$Nu = \frac{h}{\frac{\kappa}{\sqrt{A}}} = \frac{h\sqrt{A}}{\kappa}$$

$$Nu = \frac{\rho_{ice}\sqrt{A}\frac{\partial A}{\partial t}(L_f + c_s T_i)}{\kappa T_{water} P}$$

Melting under convection

Rayleigh number

$$Ra = \frac{g\Delta\rho(2R)^3}{\alpha\nu\bar{\rho}}$$



Yamada et al., IJHMT, 1997

$$Nu_m^* = 8.05 \times 10^{-2} Ra^{0.32} \quad (3 \times 10^7 \leqslant Ra \leqslant 10^9).$$

$$Nu^{1/2} = 0{\cdot}60 + 0{\cdot}387 \left(\frac{Ra}{[1 + (0{\cdot}559/Pr)^{9/16}]^{16/9}} \right)^{1/6}. \quad (10)$$

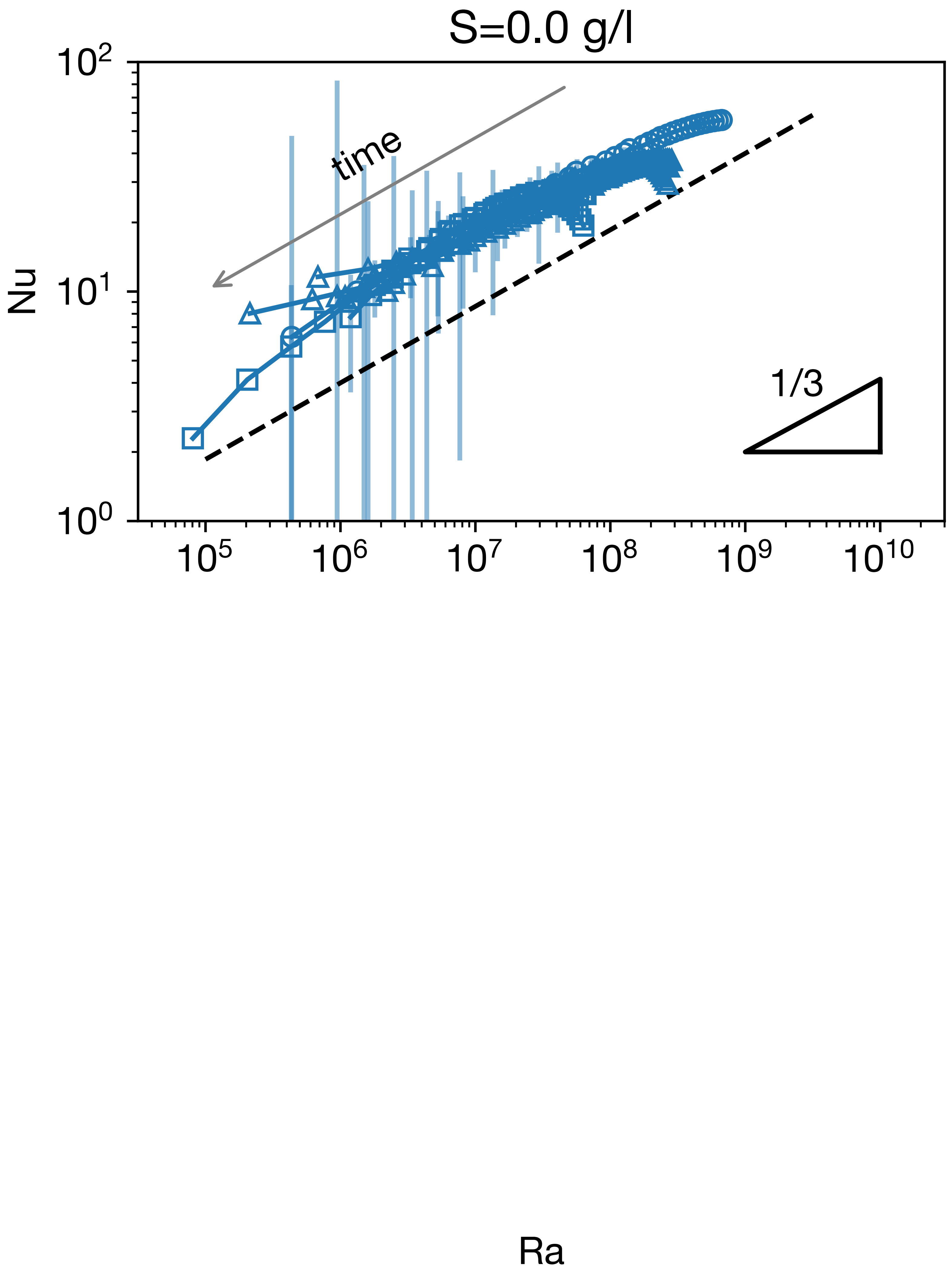
Churchill and Chu, IJHMT, 1975

$$\mathrm{Nu}_d = \frac{h_m d}{k_w} = C_1 \mathrm{Ra}_w^{1/3} \frac{A_w}{A} + C_2 \mathrm{Ra}_a^{1/3} \frac{A_a}{A} \left(\frac{\Delta T_a}{\Delta T_w} \right) \left(\frac{k_a}{k_w} \right). \quad (40)$$

Hosseini and Rahaeifard, Exp. Heat Trans., 2009

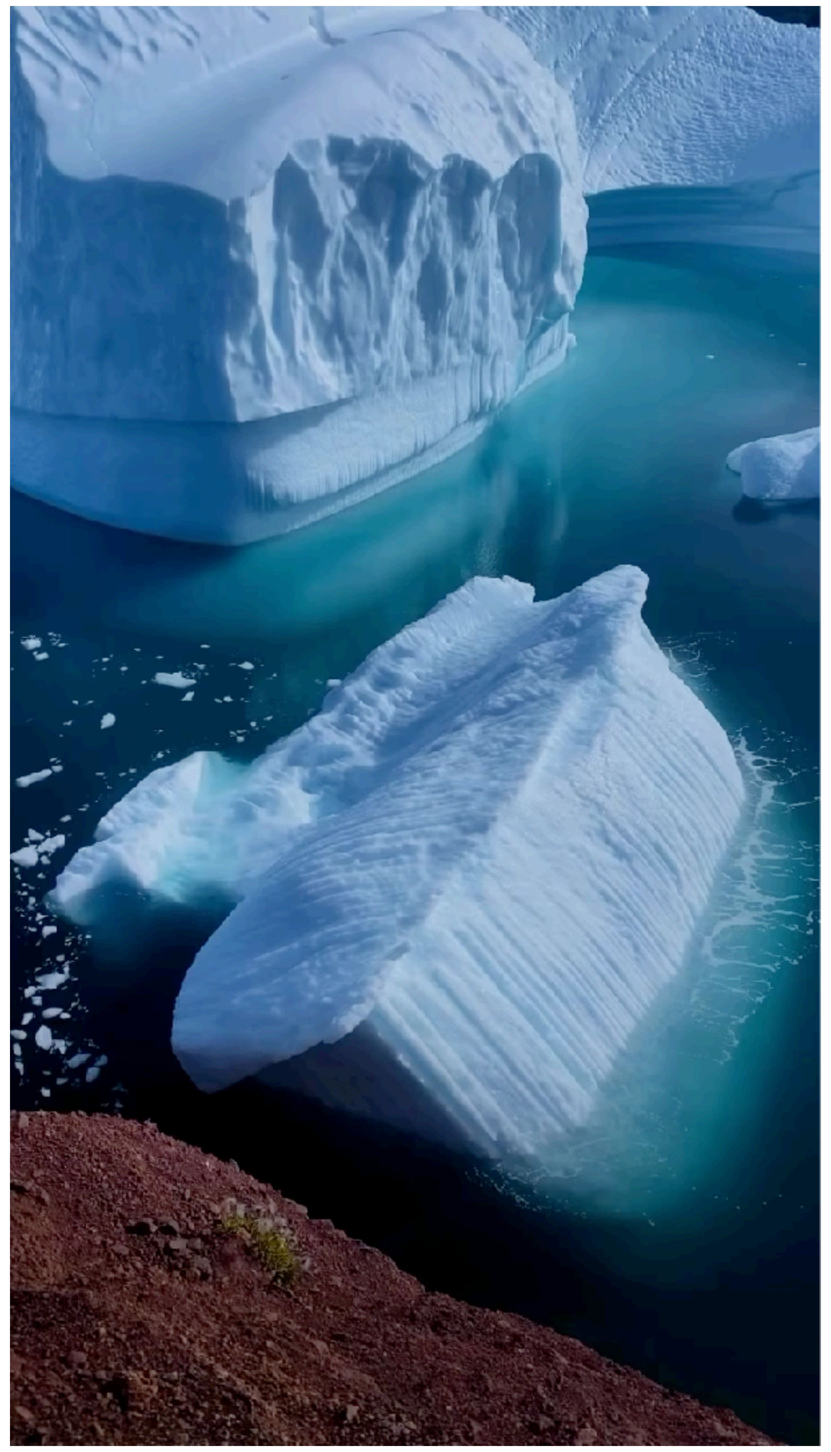
Melting under convection

Nusselt - Rayleigh scaling



Summary and conclusions

Summary



Putting everything together

$$\frac{dR}{dt} = \frac{kT_0}{\rho_s \mathscr{L}} \left\{ \frac{1}{R} + \frac{1}{\sqrt{\pi \alpha t}} \right\}$$

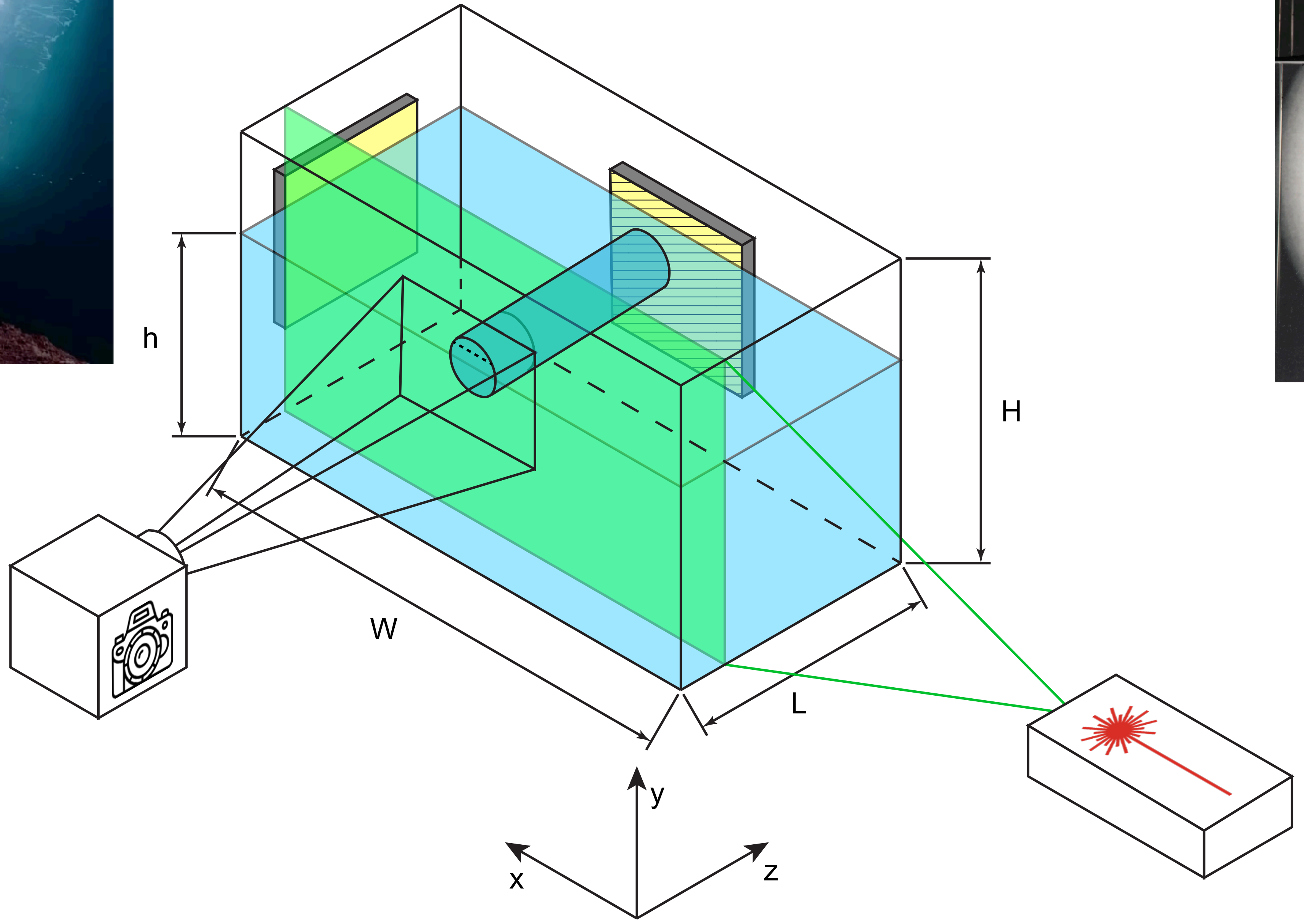
Advection

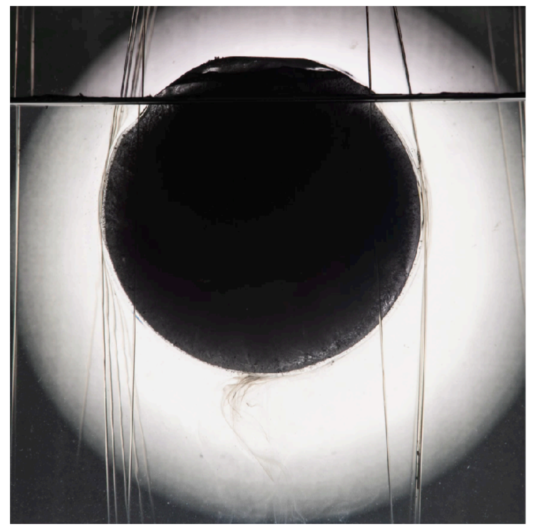
Convection

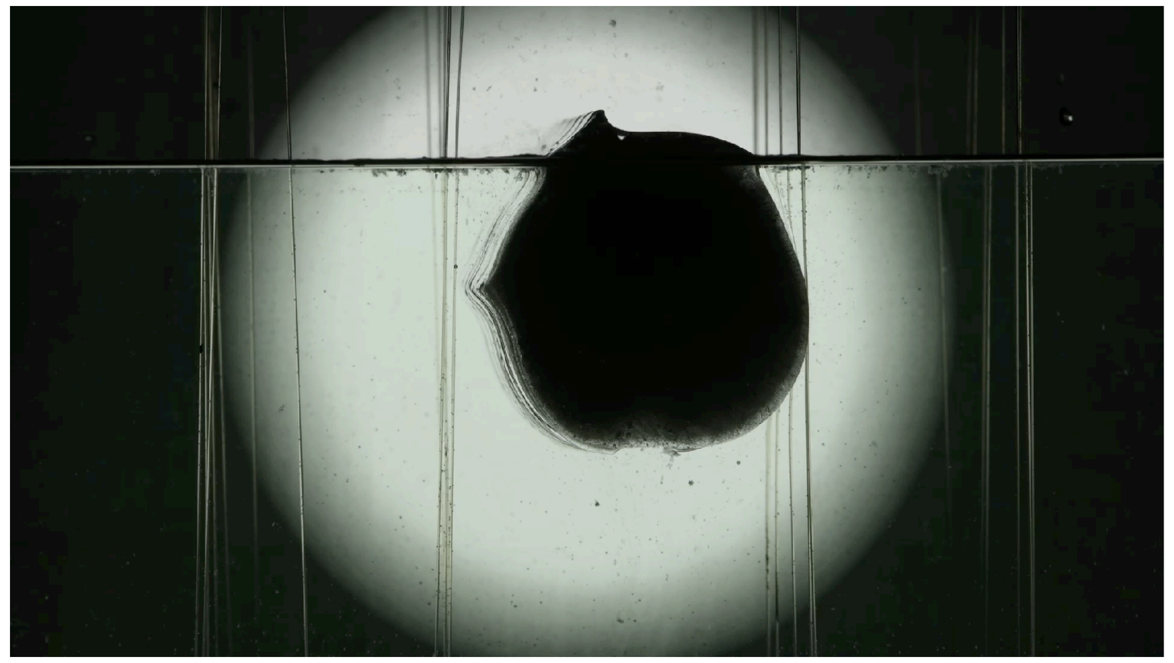
Conduction in solid

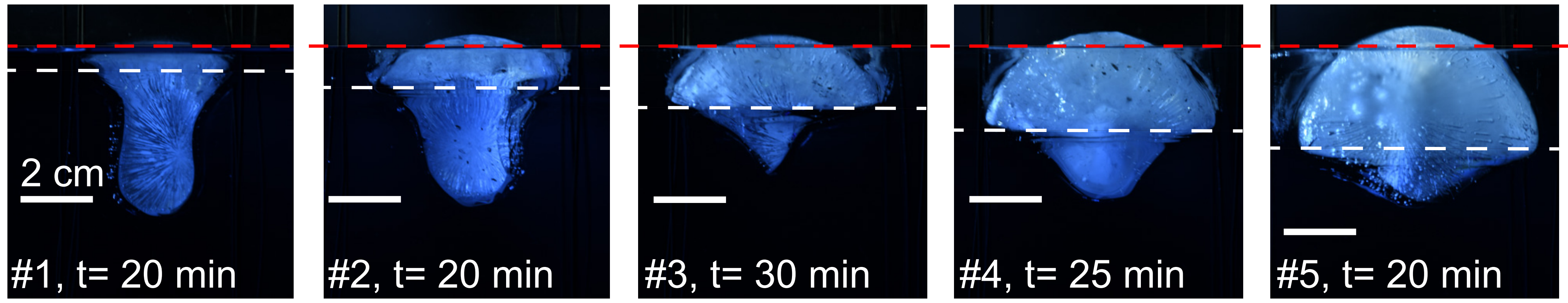
Solutes

...

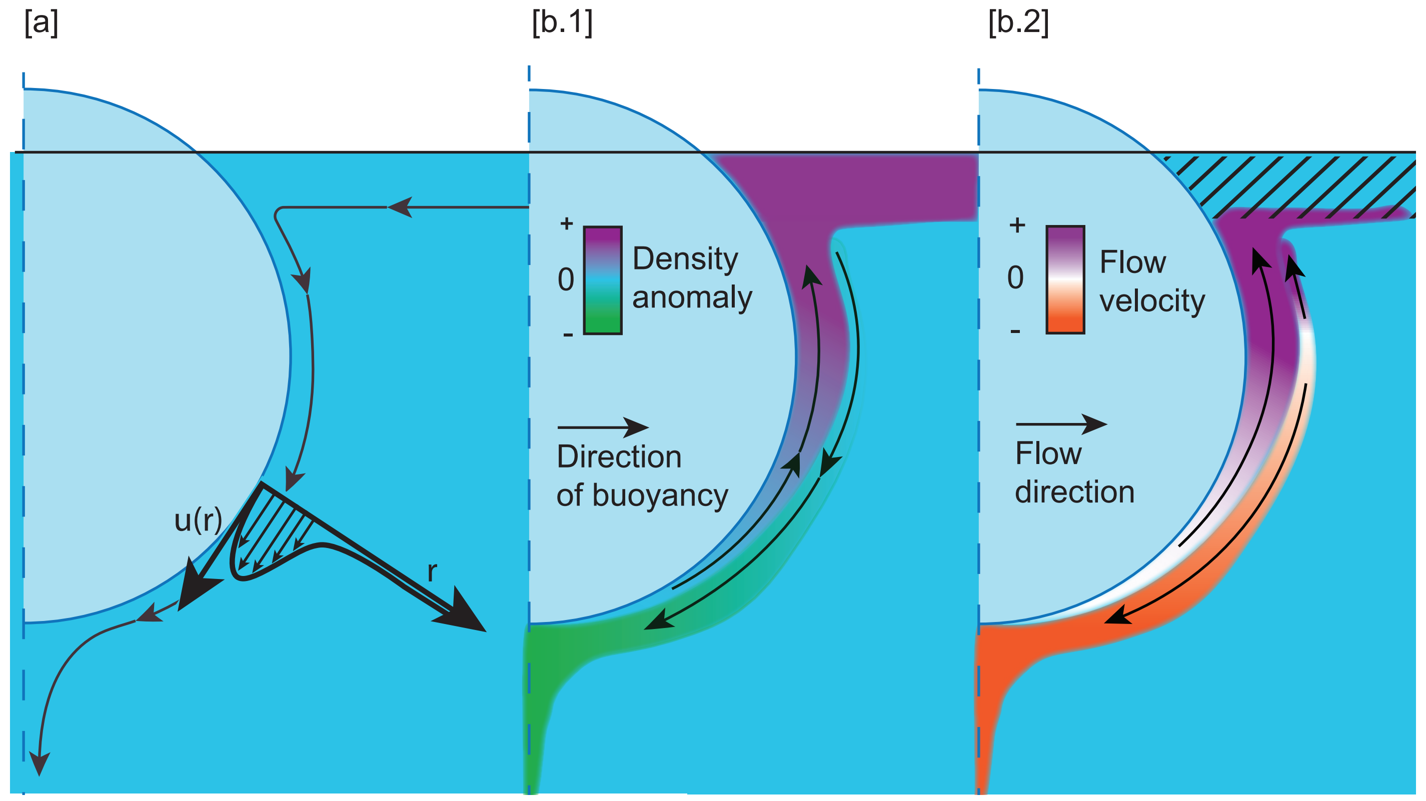


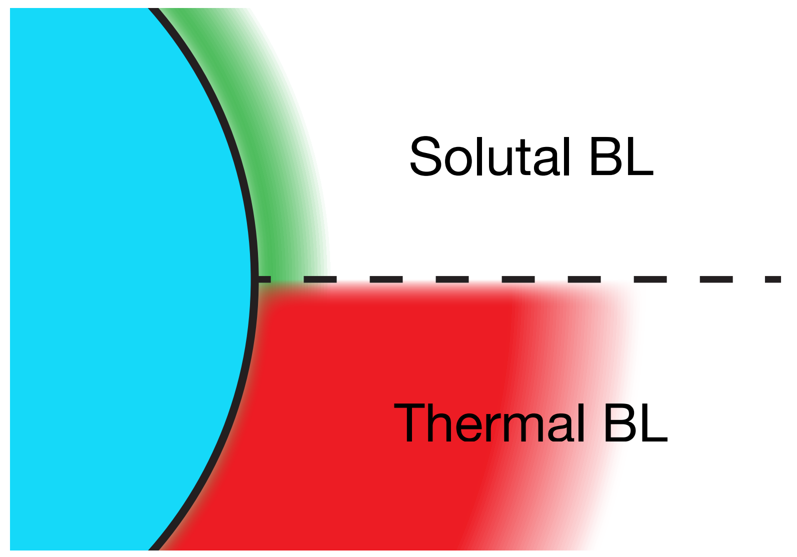


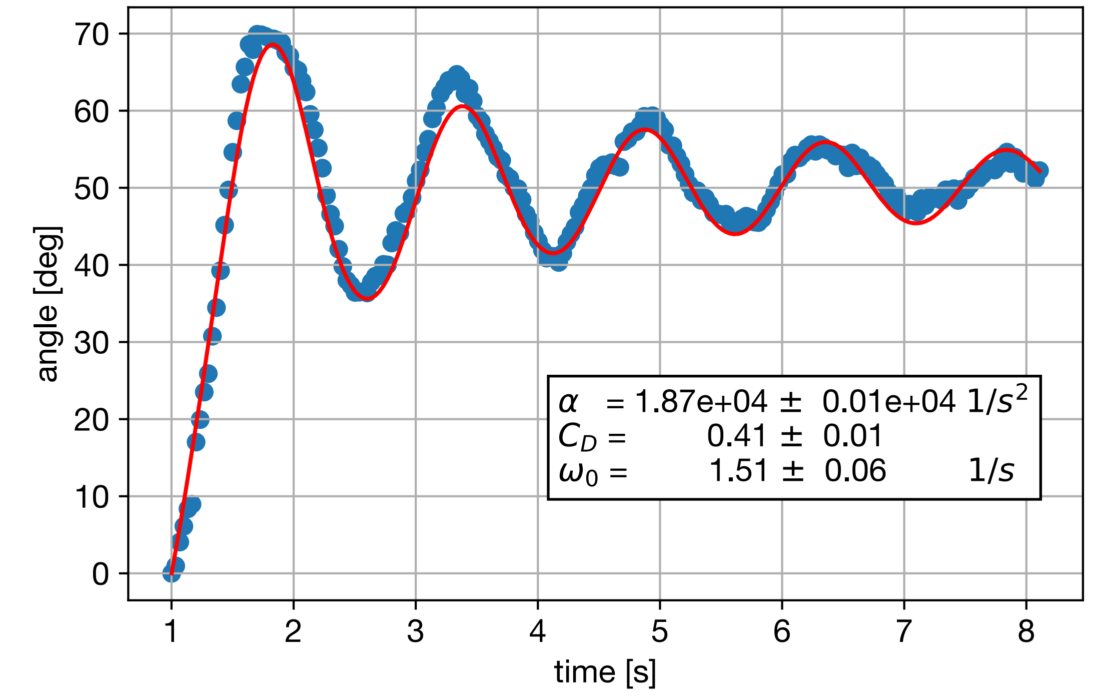


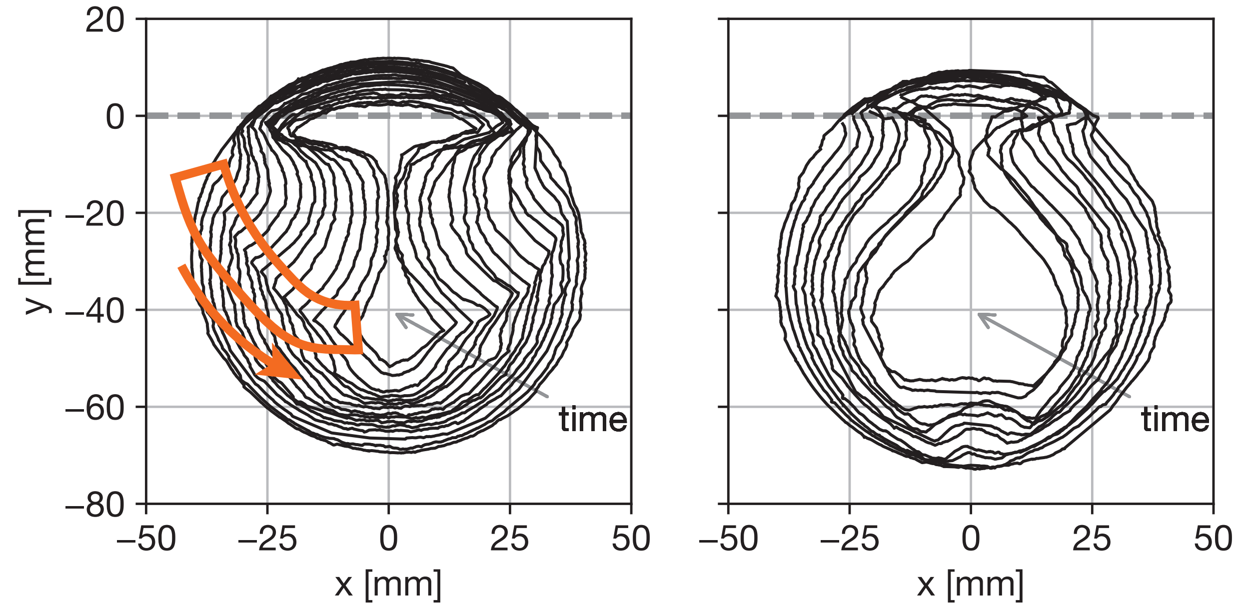


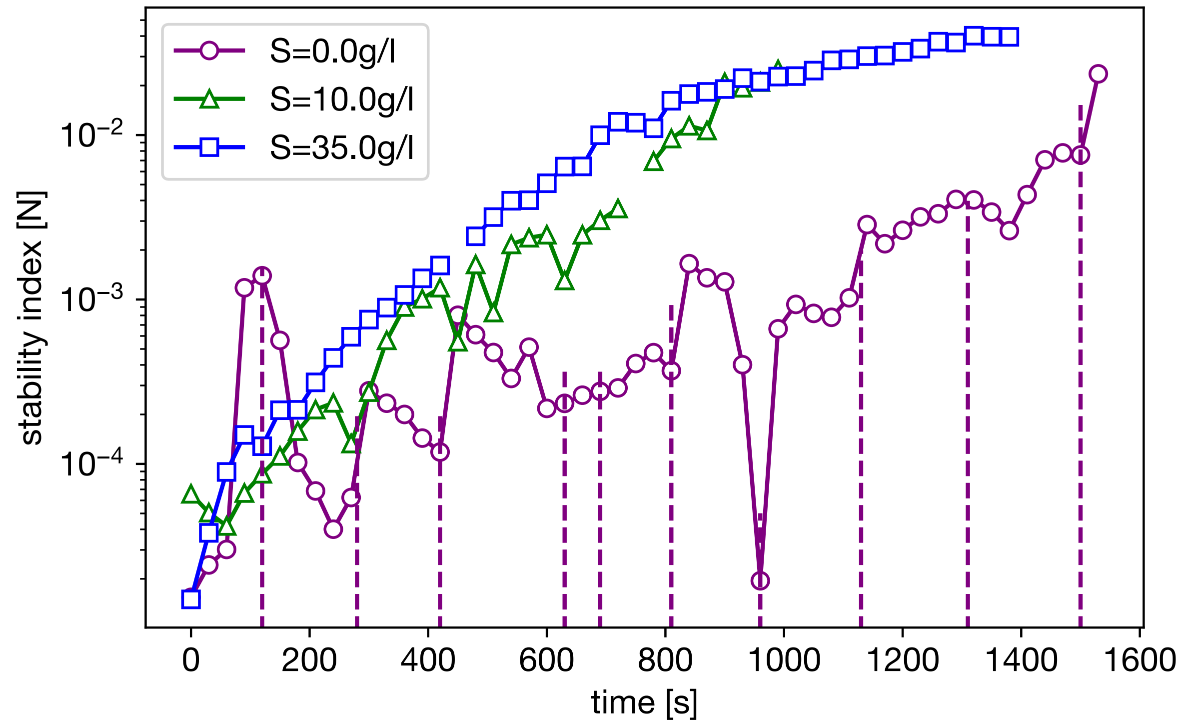
Results

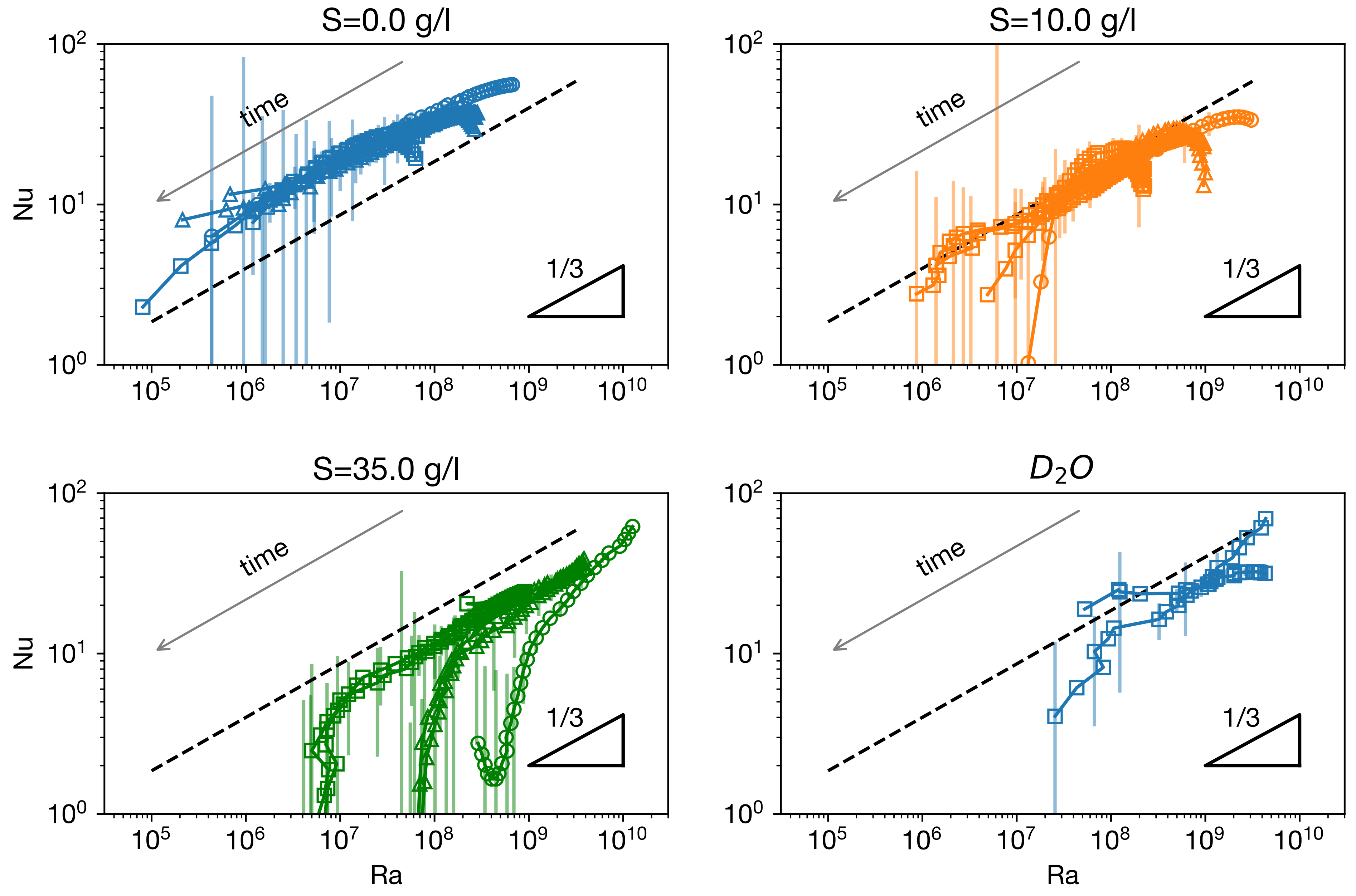

[a]
[b.1]
[b.2]
u(r)
r
Density anomaly
Direction of buoyancy
Flow velocity
Flow direction

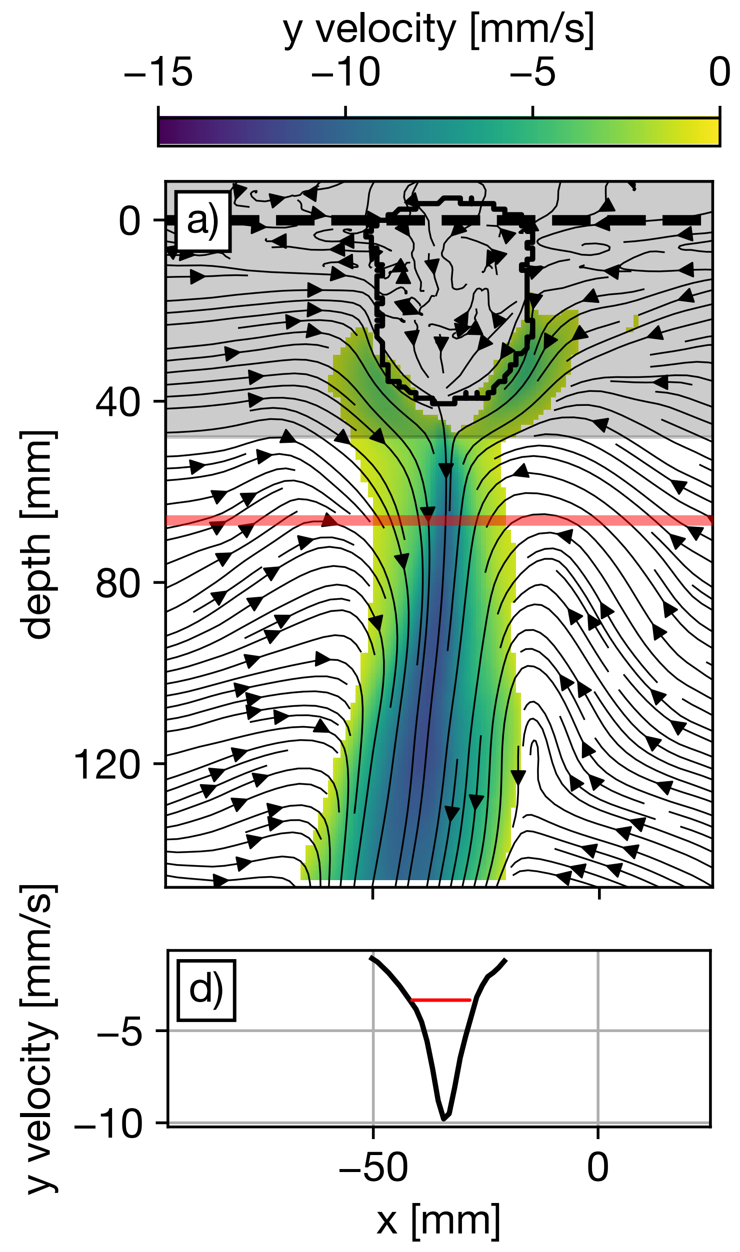

Solutal BL
Thermal BL

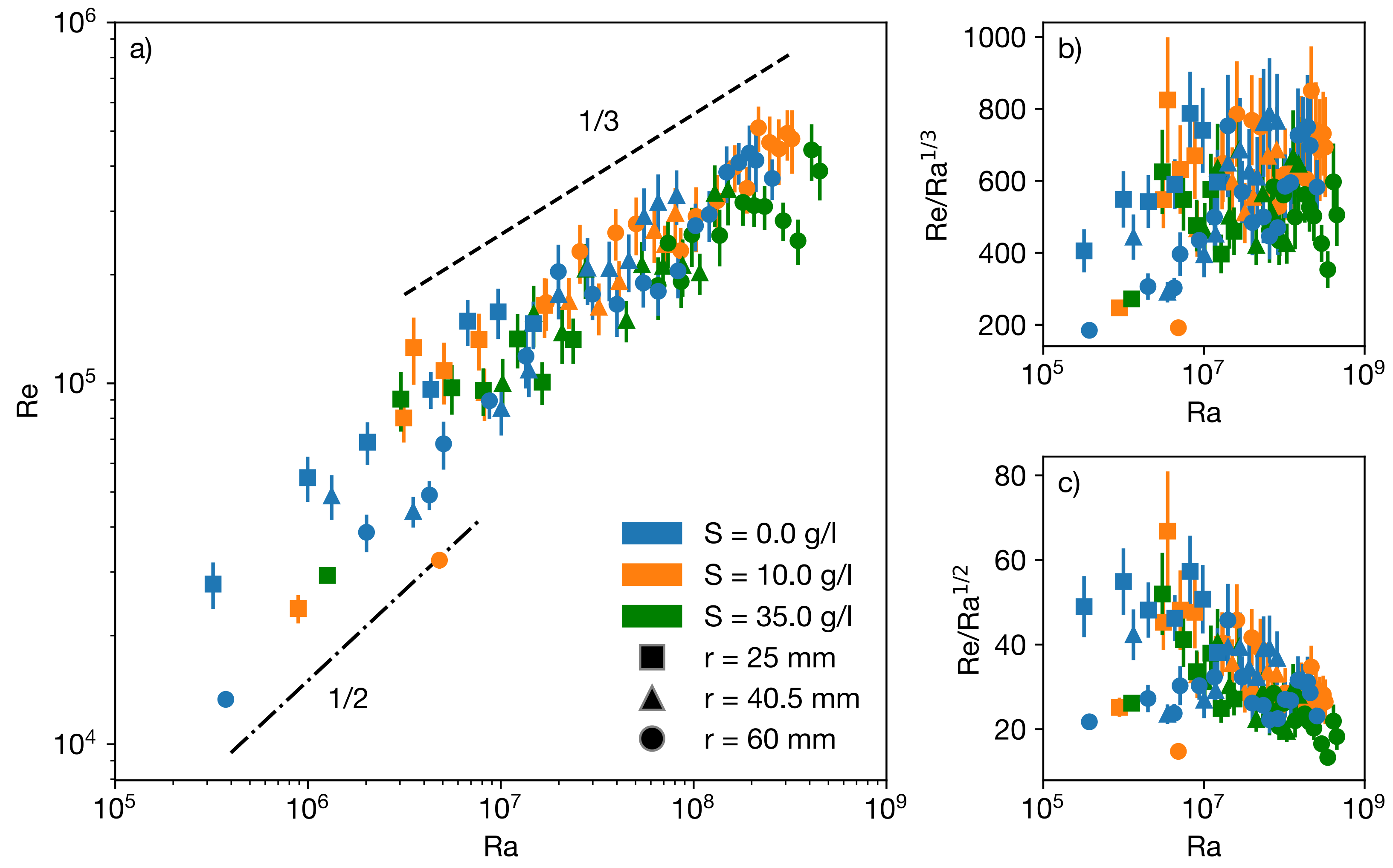

angle [deg]
time [s]
α = 1.87e+04 ± 0.01e+04 $1/s^2$
C_D = 0.41 ± 0.01
ω_0 = 1.51 ± 0.06 1/s


y [mm]
x [mm]
time
x [mm]
time


S=0.0g/l
S=10.0g/l
S=35.0g/l
stability index [N]
time [s]


S=0.0 g/l
S=10.0 g/l
S=35.0 g/l
D_2O
Nu
Ra
time
1/3


y velocity [mm/s]
a)
depth [mm]
d)
y velocity [mm/s]
x [mm]


a)
Re
Ra
1/3
1/2
S = 0.0 g/l
S = 10.0 g/l
S = 35.0 g/l
r = 25 mm
r = 40.5 mm
r = 60 mm
b)
$Re/Ra^{1/3}$
c)
$Re/Ra^{1/2}$

Conclusions

Laboratory icebergs?

- Aquarium -> Fjords
  - Confined basin, stratified polar and Atlantic waters
  - 0-50% immersion of icebergs in AW [1,2]
- Accumulation of the plume around the iceberg [3]
  - Different melting (0°C water) and diffusion timescales
  - Keel depth? Density anomaly close to iceberg?

[1] FitzMaurice et al., Geophys. Res. Lett., 2016

[2] Jackson et al., Nat. Geosci., 2014

[3] Yankovsky & Yashayaev, DEEP-SEA RES PT I, 2014

Prospects

For all the melting problems

- Understanding of boundary layer interactions
- (KH?) instability and scallops

Thank you!

Extra slides

More mathematics

Spherical coordinates

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})$$
$$= \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}$$

$$\nabla^2 u = \nabla^2 \frac{v}{r} = \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)\frac{v}{r} =$$

$$\frac{2}{r}\frac{\partial}{\partial r}\frac{v}{r} = -v\frac{2}{r^3} + \frac{2}{r^2}\frac{\partial v}{\partial r}$$

$$\frac{\partial^2}{\partial r^2}\frac{v}{r} = \frac{\partial}{\partial r}\left(-\frac{v}{r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right) =$$

$$= -\frac{1}{r^2}\frac{\partial v}{\partial r} + \frac{2}{r^3}v - \frac{1}{r^2}\frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial^2 v}{\partial r^2}$$

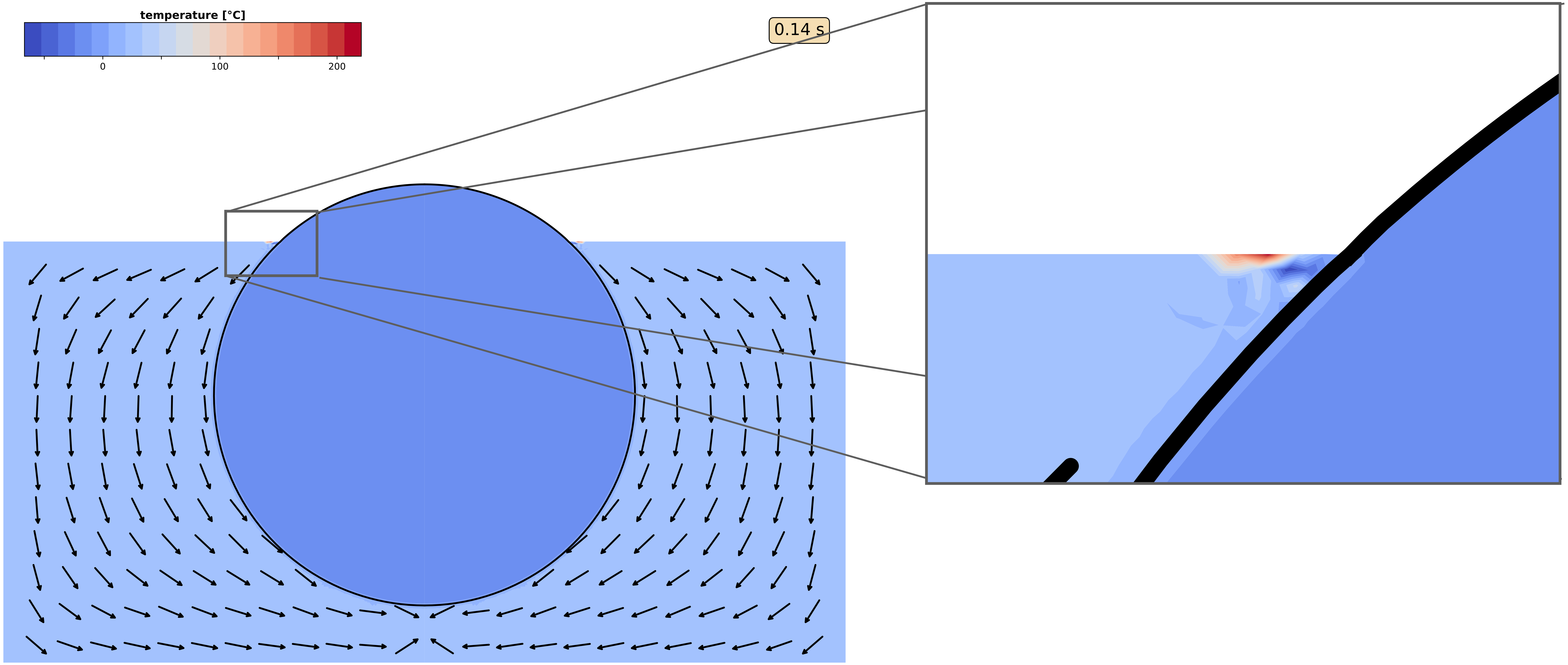
$$= \frac{2}{r^3}v - \frac{2}{r^2}\frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial^2 v}{\partial r^2}$$

Cylindrical coordinates

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})$$
$$= \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

Numerics

Finite elements (pyoomph)

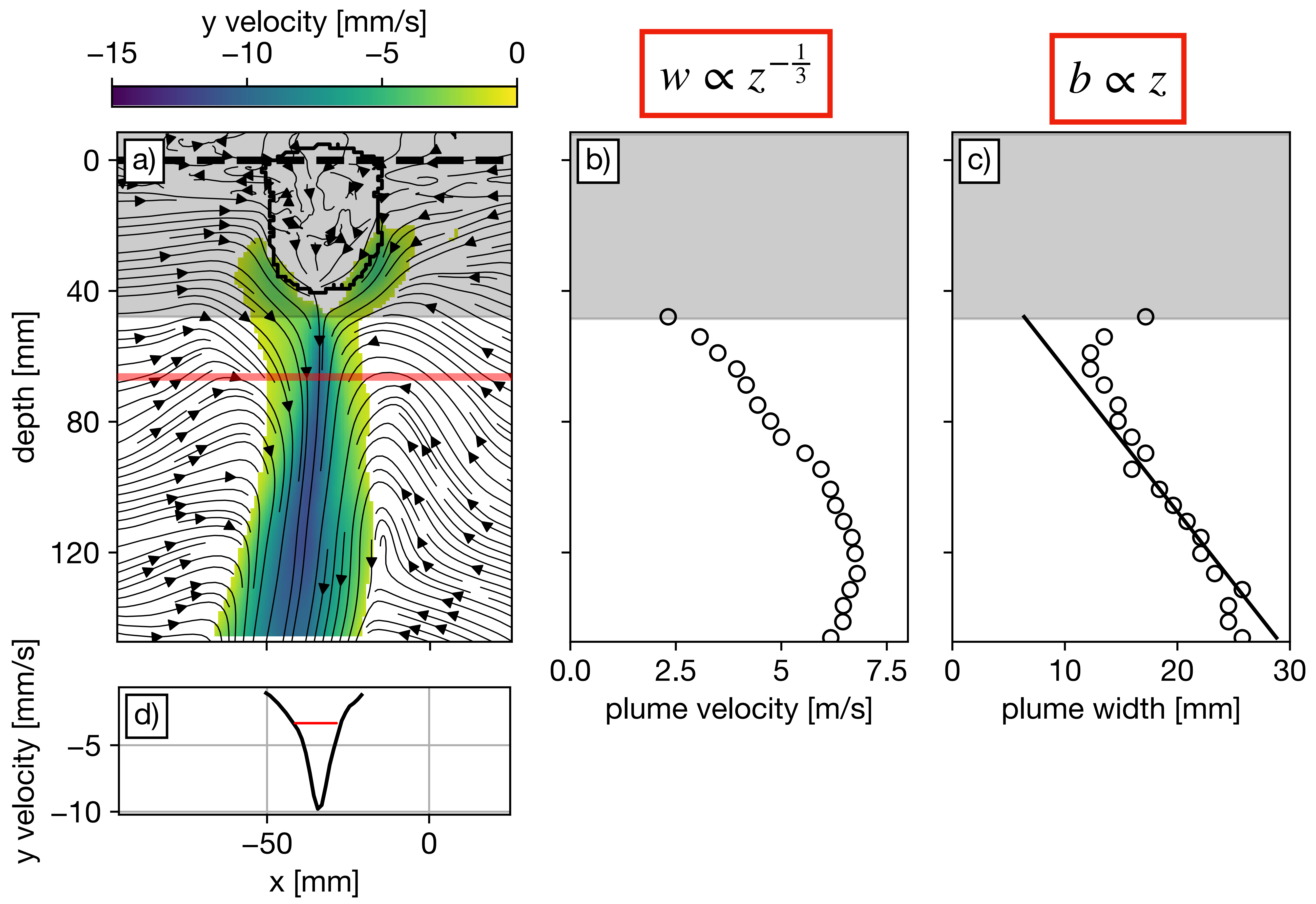


Numerics

Basilisk

Sinking plume

Reynolds number



Morton, Taylor, Turner,

JFM 1956

$$b = \frac{6}{5}\alpha z$$

$$w = \frac{5}{6\alpha}\left(\frac{9}{10}\alpha B\right)^{\frac{1}{3}}\pi^{-\frac{1}{3}}z^{-\frac{1}{3}}$$